Case Study

The following is a case study showing the resolution of the problem using SDDP and hanging branches method.

Data of the system

The following model has 1 hydro generator with storage, 1 thermal generator and 1 node. Each unit has a max capacity equal to 100 MW.

The operational costs (C) are 1 \$/MWh for the thermal generator and 0 \$/MWh for the hydro generator. The unserved energy cost (USE) is 10 \$/MWh.

Table 3 summarizes generators data.

Table 3: Generator technical data

Generator	Name	Node	MSL (MW)	Max Capacity (MW)
Thermal	G_{th}	Gens	0	100
Hydro	G_h	Gens	0	100

Table 4 summarizes generator cost parameters.

Table 4: Generator costs parameters

Name	C (\$/MWh)
C_{th}	1
C_h	0
C_{USE}	10

The horizon is segmented into 3 blocks. The first two blocks have 1 week duration and the third block has two weeks duration. The loads (D) are 90, 160 and 110 MW for blocks 1, 2 and 3 respectively.

The initial volume (v_0) of the storage is 60.48 Mm^3 and its max capacity is 100 Mm^3. The storage has recycle end effects with a penalty cost equal to 1.5 times unserved energy cost (1.5 USE). The hydro generator has 1 $MW/m^3/s$ efficiency. The inflow is 50 m^3/s for the first block; then there are 3 inflow possibilities for the second stage: 10, 50 or 90 m^3/s and the same 3 inflow possibilities for the last stage.

Table 5 and Table 6 summarize the additional input information.

Table 5: Storage properties

Min Vol (Mm3)	Max Vol (Mm3)	ρ ($MW/m^3/s$)
60.48	100	1

Table 6: General Information per block

<table>
<tr><th>Stages (t)</th><th>1</th><th>2</th><th>3</th></tr>
<tr><td>Duration (h)</td><td>168</td><td>168</td><td>336</td></tr>
<tr><td>Load (MW)</td><td>90</td><td>160</td><td>110</td></tr>
<tr><td rowspan="3">Inflow (m3/s)</td><td rowspan="3">50</td><td>10</td><td>40</td></tr>
<tr><td>50</td><td>50</td></tr>
<tr><td>90</td><td>60</td></tr>
</table>

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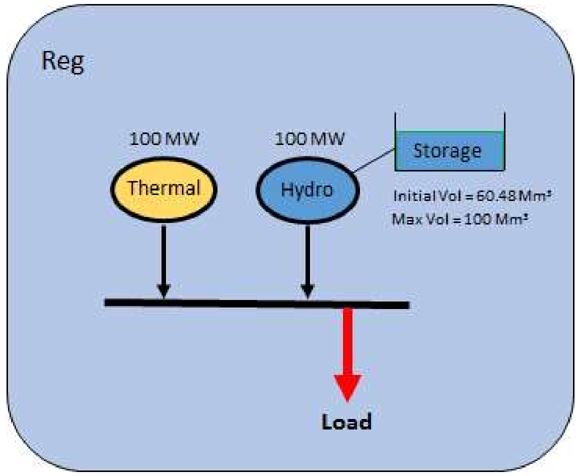

Figure 26. Representation of the Power System

The optimization problem determines the optimal dispatch of the system that minimizes production costs given inflows uncertainty. Each decision is re-evaluated at the beginning of each block when new inflow forecast arrives to the decision maker.

The problem is a stochastic multi-stage optimization problem that can be represented in mathematical terms as shown in equation (13) where each block represents a stage:

$$Z = min\left[\sum_{t=1}^{H}\sum_{k=1}^{K(t)} p_{t,k} \cdot \Delta t \cdot \left(C\_th \cdot G\_th_{t,k} + C\_USE \cdot USE_{t,k}\right)\right] \tag{13}$$

Subject to:

$$G\_h_{t,k} + G\_th_{t,k} + USE_{t,k} = D_t$$
$$G\_h_{t,k} = \rho \cdot q_{t,k}$$
$$v_{t,k} = v_{t-1,k} + I - R$$
$$\begin{bmatrix} G\_th_{min} \\ G\_h_{min} \\ 0 \\ v_{min} \end{bmatrix} \leq x = \begin{bmatrix} G\_th_{t,k} \\ G\_h_{t,k} \\ USE_{t,k} \\ v_{t,k} \end{bmatrix} \leq \begin{bmatrix} G\_th_{max} \\ G\_h_{max} \\ D_t \\ v_{max} \end{bmatrix}$$

Where:

$p_{t,k}$: Probability of sub-problem k occurring in stage t. Note that $\sum_{k=1}^{K} p_k = 1$
Δt: Duration of the stage
C_th: Cost of thermal generator per MWh
C_USE: cost of unserved energy per MWh
$G\_th_{t,k}$: Thermal generation of sub-problem k at stage t
$G\_h_{t,k}$: Hydro generation of sub-problem k at stage t
$USE_{t,k}$: Unserved energy of sub-problem k at stage t
$v_{t,k}$: Stored end volume of sub-problem k at stage t
$v_{t-1,k}$: Stored end volume of sub-problem k at stage t-1
D_t: Load in stage t
I: Inflow ($I = u \cdot A_{t,k}$)
R: Release ($R = u \cdot q_{t,k}$)
u: Hydro factor, refer to equation (14)
$A_{t,k}$: Inflow (m3/s) of sub-problem k at stage t

Equation (14) shows the value for factor "u" to convert hydro inflows into cubic meters.

$$u = u(\Delta t) = 3600\left[\frac{s}{h}\right]\cdot \Delta t[h] = 0.0036[Ms] \tag{14}$$

Because the stages have different durations then equation (14) takes the following values:

$$u_{1,2} = u(168) = 0.6048$$
$$u_3 = u(336) = 1.2096$$

By using the equations above, an inflow equal to 50 m^3/s stores 30.24 Mm^3 of water in the first and second stages, and 60.48 Mm^3 in the last stage.

1 MWh deviation in the end effect recycle condition is equal to the following cost per Mm^3:

$$\left[\frac{\$}{Mm^3}\right] = \left[\frac{\$}{MWh}\right]\cdot \rho\left[\frac{MW}{m^3/s}\right]\cdot \frac{10^6}{3600[s/h]} \tag{15}$$

$$= 1.5 \cdot 10 \cdot 1 \cdot 277.78\ [\$/Mm^3]$$
$$=1.5 \cdot 2777.78\ [\$/Mm^3]$$

This value can be used to create a Future Cost Function (FCF) at the end of the planning horizon. The constraint is built using equation (16).

$$\propto_{t,k} \geq \alpha^*_{t,k} + \pi^*_{t,k}(v_{t,k} - v^*_{t-1,k}) \tag{16}$$

Equation (16) takes the following values

$$\alpha_{3,k} \geq 0 - 1.5\cdot 2777.78 \cdot \left(v_{3,k} - 60.48\right) = 4166.67\cdot\left(60.48 - v_{3,k}\right)$$

Equation (13) can be rewritten in the following extensive form

$$Z = \min\left[G\_th_{1,1} + 10\cdot USE_{1,1} + \frac{1}{3}\sum_{k=1}^{3}\left[G\_th_{2,k} + 10\cdot USE_{2,k}\right] + \frac{1}{9}\sum_{k=1}^{9}\left[G\_th_{3,k} + 10\cdot USE_{3,k}\right]\right]$$
$$G\_h_{1,1} + G\_th_{1,1} + USE_{1,1} = 90$$
$$G\_h_{2,1} + G\_th_{2,1} + USE_{2,1} = 160$$
$$G\_h_{2,2} + G\_th_{2,2} + USE_{2,2} = 160$$
$$G\_h_{2,3} + G\_th_{2,3} + USE_{2,3} = 160$$
$$G\_h_{3,1} + G\_th_{3,1} + USE_{3,1} = 110$$
$$G\_h_{3,2} + G\_th_{3,2} + USE_{3,2} = 110$$
$$G\_h_{3,3} + G\_th_{3,3} + USE_{3,3} = 110$$
$$G\_h_{3,4} + G\_th_{3,4} + USE_{3,4} = 110$$
$$G\_h_{3,5} + G\_th_{3,5} + USE_{3,5} = 110$$
$$G\_h_{3,6} + G\_th_{3,6} + USE_{3,6} = 110$$
$$G\_h_{3,7} + G\_th_{3,7} + USE_{3,7} = 110$$
$$G\_h_{3,8} + G\_th_{3,8} + USE_{3,8} = 110$$
$$G\_h_{3,9} + G\_th_{3,9} + USE_{3,9} = 110$$
$$v_{1,1} = v_0 + 30.24 - 0.6048\cdot G\_h_{1,1}$$
$$v_{2,1} = v_{1,1} + 6.048 - 0.6048\cdot G\_h_{2,1}$$
$$v_{2,2} = v_{1,1} + 30.24 - 0.6048\cdot G\_h_{2,2}$$
$$v_{2,3} = v_{1,1} + 54.432 - 0.6048\cdot G\_h_{2,3}$$
$$v_{3,1} = v_{2,1} + 48.384 - 1.2096\cdot G\_h_{3,1}$$
$$v_{3,2} = v_{2,1} + 60.48 - 1.2096\cdot G\_h_{3,2}$$
$$v_{3,3} = v_{2,1} + 72.576 - 1.2096\cdot G\_h_{3,3}$$
$$v_{3,4} = v_{2,2} + 48.384 - 1.2096\cdot G\_h_{3,4}$$

$$v_{3,5} = v_{2,2} + 60.48 - 1.2096 \cdot G\_h_{3,5}$$
$$v_{3,6} = v_{2,2} + 72.576 - 1.2096 \cdot G\_h_{3,6}$$
$$v_{3,7} = v_{2,3} + 48.384 - 1.2096 \cdot G\_h_{3,7}$$
$$v_{3,8} = v_{2,3} + 60.48 - 1.2096 \cdot G\_h_{3,8}$$
$$v_{3,9} = v_{2,3} + 72.576 - 1.2096 \cdot G\_h_{3,9}$$
$$\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix} \le x = \begin{bmatrix}G\_th_{t,k}\\G\_h_{t,k}\\USE_{1,k}\\USE_{2,k}\\USE_{3,k}\\v_{t,k}\end{bmatrix} \le \begin{bmatrix}100\\100\\90\\160\\110\\100\end{bmatrix}$$

(17)

Model solution using SDDP algorithm.

Figure 27 illustrates the decision tree of the stochastic problem. It shows the sub-problems (t,k) together with the inflow data ($A_{t,k}$).

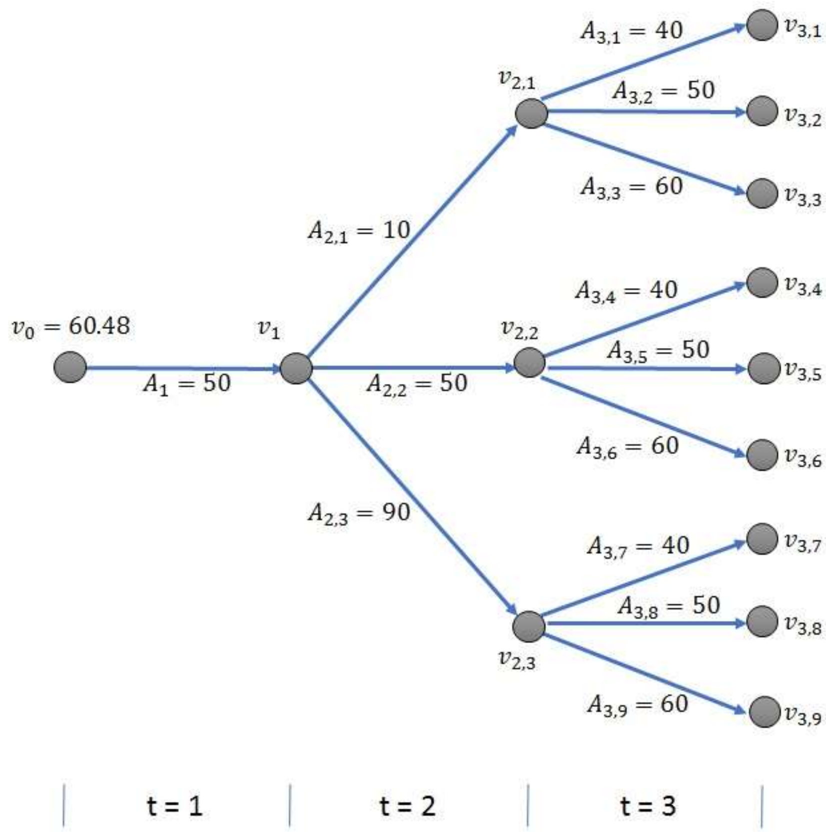

Figure 27. Decision Tree

When this system is solved using SDDP algorithm, 4 sub-problems are solved in forward pass (1 on the first stage and 3 on the second one) and 12 sub-problems are solved in backward pass (9 on the last stage and 3 on the second one). Equation (18) shows the sub-problem to be solved in each iteration.

$$Z_{t,k}(v_{t-1,k}) = min[\Delta t \cdot (C\_th \cdot G\_th_{t,k} + C\_USE \cdot USE_{t,k}) + \alpha_{t,k}] \quad (18)$$

$$G\_h_{t,k} + G\_th_{t,k} + USE_{t,k} \geq D^t$$
$$G\_h_{t,k} = \rho \cdot q_{t,k}$$
$$v_{t,k} = v_{t-1,k} + I - R \qquad \rightarrow \pi_t^*$$
$$\alpha_{t,k} \geq 0$$
$$\begin{bmatrix} G\_h_{min} \\ G\_th_{min} \\ 0 \\ v_{min} \end{bmatrix} \leq \begin{bmatrix} G\_h_{t,k} \\ G\_th_{t,k} \\ USE_{t,k} \\ v_{t,k} \end{bmatrix} \leq \begin{bmatrix} G\_h_{max} \\ G\_th_{max} \\ USE_t \\ v_{max} \end{bmatrix}$$

Where:

α: variable representing the expected future cost value of the following stage sub-problem

ITERATION 1

Direction: Forward Pass

Equation (18) takes the following form for the stage 1 on forward pass in iteration 1.

$$Z_1(0) = min[G\_th_1 + 10USE_1 + \alpha_1]$$
$$G\_th_1 + G\_h_1 + USE_1 = 90$$
$$v_1 = 90.72 - 0.6048 \cdot G\_h_1$$
$$\alpha_1 \geq 0$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_1 \\ G\_th_1 \\ USE_1 \\ v_1 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 90 \\ 100 \end{bmatrix}$$

Equation (18) takes the following forms for the stage 2 on forward pass in iteration 1.

$$Z_{2,1}(v_{1,1}) = min[G\_th_{2,1} + 10USE_{2,1} + \alpha_{2,1}]$$
$$G\_th_{2,1} + G\_h_{2,1} + USE_{2,1} = 160$$
$$v_{2,1} = v_{1,1} + u \cdot A_{2,1} - 0.6048 \cdot G\_h_{2,1}$$
$$\alpha_{2,1} \geq 0$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,1} \\ G\_th_{2,1} \\ USE_2 \\ v_2 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

$$Z_{2,2}(v_{1,1}) = min[G\_th_{2,2} + 10USE_{2,2} + \alpha_{2,2}]$$
$$G\_th_{2,2} + G\_h_{2,2} + USE_{2,2} = 160$$
$$v_{2,2} = v_{1,1} + u \cdot A_{2,2} - 0.6048 \cdot G\_h_{2,2}$$
$$\alpha_{2,2} \geq 0$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,2} \\ G\_th_{2,2} \\ USE_2 \\ v_2 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

$$Z_{2,3}(v_{1,1}) = min[G\_th_{2,3} + 10USE_{2,3} + \alpha_{2,3}]$$
$$G\_th_{2,3} + G\_h_{2,3} + USE_{2,3} = 160$$
$$v_{2,3} = v_{1,1} + u \cdot A_{2,3} - 0.6048 \cdot G\_h_{2,3}$$
$$\alpha_{2,3} \geq 0$$

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$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_h_{2,3}\\G\_th_{2,3}\\USE_3\\v_3\end{bmatrix} \leq \begin{bmatrix}100\\100\\160\\100\end{bmatrix}$$

Because in this first pass we do not have approximation for the Future Cost Function (FCF), the stored volume at the end of the stages is zero (See Figure 28). This means that all the inflows were used to generate electricity, and the remaining load was supplied using the thermal gen and perhaps incur in unserved energy.

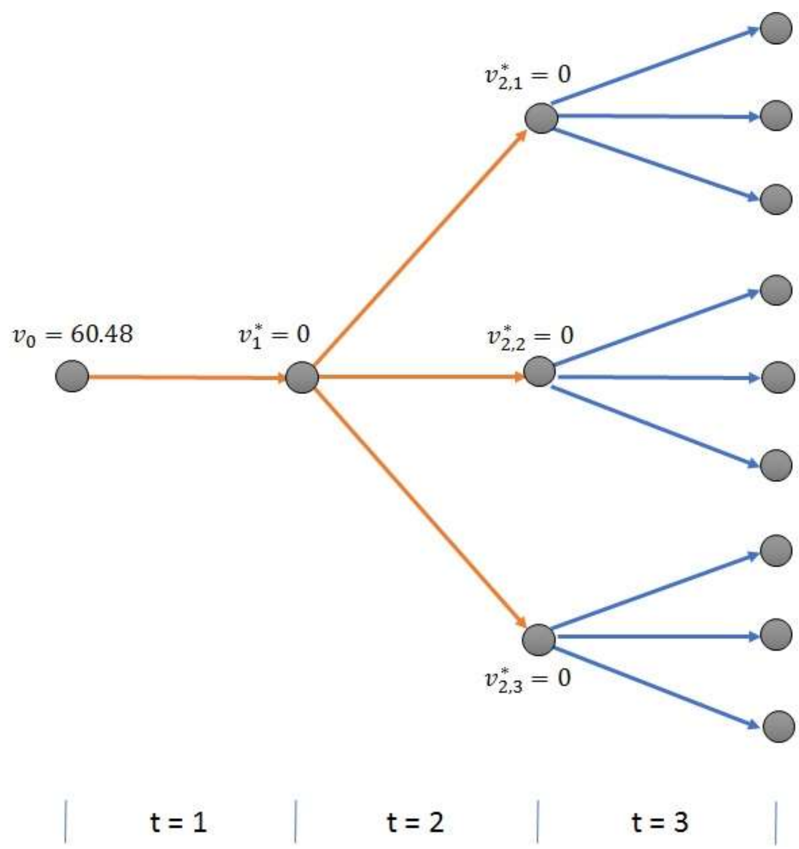


Figure 28. Iteration 1 Forward Pass.

The results for each sub-problems are presented below. Table 7 summarizes the costs of each sub-problem.

$$x_{1,1} = [G\_h_{1,1}, G\_th_{1,1}, USE_{1,1}, v_{1,1}]$$
$$x_{1,1} = [90, 0, 0, 36.29]$$
$$Z_{1,1}(0) = \ 0 + 10 \cdot 0 = \$0$$
$$x_{2,1} = [G\_h_{2,1}, G\_th_{2,1}, USE_{2,1}, v_{2,1}]$$
$$x_{2,1} = [70, 90, 0, 0]$$
$$Z_{2,1}(0) = 90 + 10 \cdot 0 = \$90/h \times 168h = \$15{,}120$$
$$x_{2,2} = [G\_h_{2,2}, G\_th_{2,2}, USE_{2,2}, v_{2,2}]$$
$$x_{2,2} = [100, 60, 0, 6.05]$$
$$Z_{2,2}(0) = 60 + 10 \cdot 10 = \$60/h \times 168h = \$10{,}080$$
$$x_{2,3} = [G\_h_{2,3}, G\_th_{2,3}, USE_{2,3}, v_{2,3}]$$
$$x_{2,3} = [100, 60, 0, 30.24]$$
$$Z_{2,3}(0) = 60 + 10 \cdot 0 = \$60/h \times 168h = \$10{,}080$$

Table 7: Results of Iteration 1 Forwards Pass.

t	K	$v^*_{t,k}$	AC	FCF
1	1	36.29	0	0
2	1	0	15120	0
	2	6.048	10080	0
	3	30.24	10080	0

As an example, the results of the sub-problem 1 of stage 2 means that the 10 m^3/s are used to generate 70 MW with the hydro gen and 90 MW with the thermal gen, so there is no unserved energy. Thus, the cost of the sub-problem is $100,800.

Direction: Backward Pass

Equation (18) takes the following form for the stage 3 on the backward pass of iteration 1.

$$Z_{3,1}(v_{2,1}) = min[G\_th_{3,1} + 10 \cdot USE_{3,1} + \alpha_{3,1}]$$
$$G\_th_{3,1} + G\_h_{3,1} + USE_{3,1} = 110$$
$$v_{3,1} = v_{2,1} + u \cdot A_{3,1} - 1.2096 \cdot G\_h_{3,1}$$
$$\alpha_{3,1} \geq 0$$
$$\alpha_{3,1} \geq 4166.67 \cdot (60.48 - v_{3,1})$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{3,1} \\ G\_th_{3,1} \\ USE_3 \\ v_3 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 110 \\ 100 \end{bmatrix}$$

$$Z_{3,2}(v_{2,1}) = min[G\_th_{3,2} + 10 \cdot USE_{3,2} + \alpha_{3,2}]$$
$$G\_th_{3,2} + G\_h_{3,2} + USE_{3,2} = 110$$
$$v_{3,2} = v_{2,1} + u \cdot A_{3,2} - 1.2096 \cdot G\_h_{3,2}$$
$$\alpha_{3,2} \geq 0$$
$$\alpha_{3,2} \geq 4166.67 \cdot (60.48 - v_{3,2})$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{3,2} \\ G\_th_{3,2} \\ USE_3 \\ v_3 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 110 \\ 100 \end{bmatrix}$$

$$Z_{3,3}(v_{2,1}) = min[G\_th_{3,3} + 10 \cdot USE_{3,3} + \alpha_{3,3}]$$
$$G\_th_{3,3} + G\_h_{3,3} + USE_{3,3} = 110$$
$$v_{3,3} = v_{2,1} + u \cdot A_{3,3} - 1.2096 \cdot G\_h_{3,3}$$
$$\alpha_{3,3} \geq 0$$
$$\alpha_{3,3} \geq 4166.67 \cdot (60.48 - v_{3,3})$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{3,3} \\ G\_th_{3,3} \\ USE_3 \\ v_3 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 110 \\ 100 \end{bmatrix}$$

$$Z_{3,4}(v_{2,2}) = min[G\_th_{3,4} + 10 \cdot USE_{3,4} + \alpha_{3,4}]$$
$$G\_th_{3,4} + G\_h_{3,4} + USE_{3,4} = 110$$
$$v_{3,4} = v_{2,2} + u \cdot A_{3,4} - 1.2096 \cdot G\_h_{3,4}$$
$$\alpha_{3,4} \geq 0$$
$$\alpha_{3,4} \geq 4166.67 \cdot (60.48 - v_{3,4})$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \le x = \begin{bmatrix}G\_h_{3,4}\\G\_th_{3,4}\\USE_3\\v_3\end{bmatrix} \le \begin{bmatrix}100\\100\\110\\100\end{bmatrix}$$

$$Z_{3,5}(v_{2,2}) = min[G\_th_{3,5} + 10 \cdot USE_{3,5} + \alpha_{3,5}]$$
$$G\_th_{3,5} + G\_h_{3,5} + USE_{3,5} = 110$$
$$v_{3,5} = v_{2,2} + u \cdot A_{3,5} - 1.2096 \cdot G\_h_{3,5}$$
$$\alpha_{3,6} \ge 0$$
$$\alpha_{3,5} \ge 4166.67 \cdot (60.48 - v_{3,5})$$
$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \le x = \begin{bmatrix}G\_h_{3,5}\\G\_th_{3,5}\\USE_3\\v_3\end{bmatrix} \le \begin{bmatrix}100\\100\\110\\100\end{bmatrix}$$

$$Z_{3,6}(v_{2,2}) = min[G\_th_{3,6} + 10 \cdot USE_{3,6} + \alpha_{3,6}]$$
$$G\_th_{3,6} + G\_h_{3,6} + USE_{3,6} = 110$$
$$v_{3,6} = v_{2,2} + u \cdot A_{3,6} - 1.2096 \cdot G\_h_{3,6}$$
$$\alpha_{3,6} \ge 0$$
$$\alpha_{3,6} \ge 4166.67 \cdot (60.48 - v_{3,6})$$
$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \le x = \begin{bmatrix}G\_h_{3,6}\\G\_th_{3,6}\\USE_3\\v_3\end{bmatrix} \le \begin{bmatrix}100\\100\\110\\100\end{bmatrix}$$

$$Z_{3,7}(v_{2,3}) = min[G\_th_{3,7} + 10 \cdot USE_{3,7} + \alpha_{3,7}]$$
$$G\_th_{3,7} + G\_h_{3,7} + USE_{3,7} = 110$$
$$v_{3,7} = v_{2,3} + u \cdot A_{3,7} - 1.2096 \cdot G\_h_{3,7}$$
$$\alpha_{3,7} \ge 0$$
$$\alpha_{3,7} \ge 4166.67 \cdot (60.48 - v_{3,7})$$
$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \le x = \begin{bmatrix}G\_h_{3,7}\\G\_th_{3,7}\\USE_3\\v_3\end{bmatrix} \le \begin{bmatrix}100\\100\\110\\100\end{bmatrix}$$

$$Z_{3,8}(v_{2,3}) = min[G\_th_{3,8} + 10 \cdot USE_{3,8} + \alpha_{3,8}]$$
$$G\_th_{3,8} + G\_h_{3,8} + USE_{3,8} = 110$$
$$v_{3,8} = v_{2,3} + u \cdot A_{3,8} - 1.2096 \cdot G\_h_{3,8}$$
$$\alpha_{3,8} \ge 0$$
$$\alpha_{3,8} \ge 4166.67 \cdot (60.48 - v_{3,8})$$
$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \le x = \begin{bmatrix}G\_h_{3,8}\\G\_th_{3,8}\\USE_3\\v_3\end{bmatrix} \le \begin{bmatrix}100\\100\\110\\100\end{bmatrix}$$

$$Z_{3,9}(v_{2,3}) = min[G\_th_{3,9} + 10 \cdot USE_{3,9} + \alpha_{3,9}]$$
$$G\_th_{3,9} + G\_h_{3,9} + USE_{3,9} = 110$$
$$v_{3,9} = v_{2,3} + u \cdot A_{3,9} - 1.2096 \cdot G\_h_{3,9}$$
$$\alpha_{3,9} \ge 0$$
$$\alpha_{3,9} \ge 4166.67 \cdot (60.48 - v_{3,9})$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_h_{3,9}\\G\_th_{3,9}\\USE_3\\v_3\end{bmatrix} \leq \begin{bmatrix}100\\100\\110\\100\end{bmatrix}$$

The FCF's are calculated using the related sub-problems, as shown in Figure 29. Figure 29 also shows the resultant volumes in the last stage.

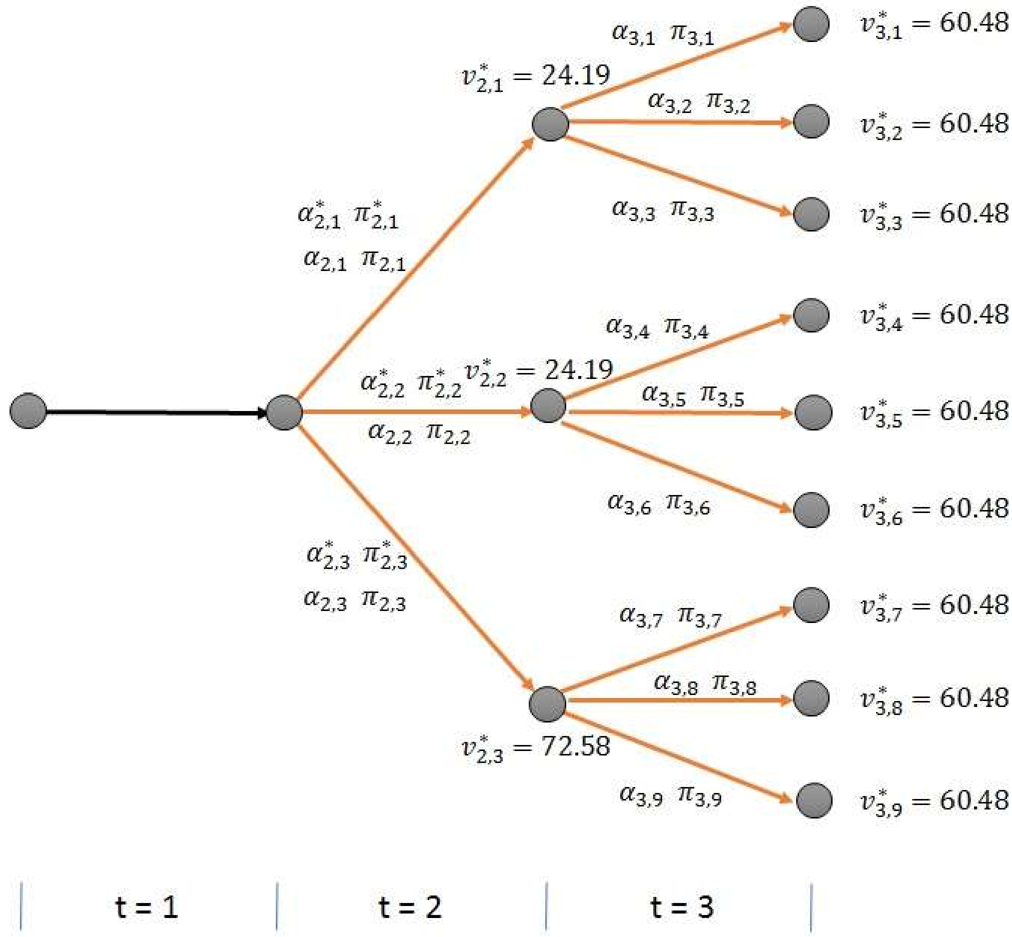


Figure 29. Iteration 1 Backward Pass.

The results for each sub-problem are as follows. Table 8 summarizes the costs of each sub-problem.

$$x_{3,1} = [0, 100, 10, 48.384]$$
$$Z_{3,1}(0) = 100 + 10 \cdot 10 = \$200/h \times 336h = \$67{,}200$$
$$x_{3,2} = [0, 100, 10, 60.48]$$
$$Z_{3,2}(0) = 100 + 10 \cdot 10 = \$200/h \times 336h = \$67{,}200$$
$$x_{3,3} = [10, 100, 0, 60.48]$$
$$Z_{3,3}(0) = 100 + 10 \cdot 0 = \$100/h \times 336h = \$33{,}600$$
$$x_{3,4} = [0, 100, 10, 54.432]$$
$$Z_{3,4}(0) = 100 + 10 \cdot 10 = \$200/h \times 336h = \$67{,}200$$
$$x_{3,5} = [5, 100, 5, 60.48]$$
$$Z_{3,5}(0) = 100 + 5 \cdot 10 = \$150/h \times 336h = \$50{,}400$$
$$x_{3,6} = [15, 95, 0, 60.48]$$
$$Z_{3,6}(0) = 95 + 10 \cdot 0 = \$95/h \times 336h = \$31{,}920$$
$$x_{3,7} = [15, 95, 0, 60.48]$$

$$Z_{3,7}(0) = 95 + 10 \cdot 0 = \$95/h \times 336h = \$31{,}920$$
$$x_{3,8} = [25, 85, 0, 60.48]$$
$$Z_{3,8}(0) = 85 + 10 \cdot 0 = \$85/h \times 336h = \$28{,}560$$
$$x_{3,9} = [35, 75, 0, 60.48]$$
$$Z_{3,9}(0) = 75 + 10 \cdot 0 = \$75/h \times 336h = \$25{,}200$$
$$x_{2,1} = [37.5, 100, 22.5, 19.66]$$
$$Z_{2,1}(0) = 100 + 10 \cdot 22.5 = \$325/h \times 168h = \$54{,}600$$
$$x_{2,2} = [60, 100, 0, 30.24]$$
$$Z_{2,2}(0) = 100 + 10 \cdot 0 = \$100/h \times 168h = \$16{,}800$$
$$x_{2,3} = [60, 100, 0, 54.432]$$
$$Z_{2,3}(0) = 100 + 10 \cdot 0 = \$100/h \times 168h = \$16{,}800$$

Table 8: Results of Iteration 1 Backward Pass

t	k	AC	FCF	$\pi_{t,k}$
3	1	67200	50400	4166.667
	2	67200	0	4166.667
	3	33600	0	2777.778
	4	67200	25200	4166.667
	5	50400	0	2777.778
	6	31920	0	277.7778
	7	31920	0	277.7778
	8	28560	0	277.7778
	9	25200	0	277.7778
2	1	54600	0	2777.778
	2	16800	0	2407.407
	3	16800	21840	277.7778

From the estimations of the third and second stage it is possible to calculate new approximations of FCF or Benders Cuts for the second and first stage, respectively.

First, it is necessary to weight the expected values of the dual variable (π) and the expected values of the optimal solution (α) according to the probability of occurrence of the sub-problem, as shown in equation (19) and equation (20).

$$\alpha^*_{t,k} = \sum_{k=1}^{K(t)} p_k \left(AC_{t+1,k} + FCF_{t+1,k} \right) \tag{19}$$

$$\pi^*_{t,k} = \sum_{k=1}^{K(t)} p_k \pi_{t+1,k} \tag{20}$$

Since the volumes obtained for each second stage sub-problem are different, then three different cuts are calculated for the second stage, using the corresponding sub-problem of stage three. Equation (19) takes the following values

$$\alpha^*_{2,1} = \left(\frac{1}{3}\right) \cdot (2 \cdot 67{,}200 + 33{,}600 + 50{,}400) = 72{,}800$$
$$\alpha^*_{2,2} = \left(\frac{1}{3}\right) \cdot (67{,}200 + 50{,}400 + 31{,}920 + 25{,}200) = 58{,}240$$

$$\alpha_{2,3}^{*} = \left(\frac{1}{3}\right) \cdot (31{,}920 + 28{,}560 + 25{,}200) = 28{,}560$$

$$\alpha_{1}^{*} = \left(\frac{1}{3}\right) \cdot (54{,}600 + 2 \cdot 16{,}800 + 21{,}840) = 36{,}680$$

Equation (20) takes the following values

$$\pi_{2,1}^{*} = \left(\frac{1}{3}\right) \cdot (2 \cdot 4{,}166.67 + 2777.78) = 3{,}703.70$$

$$\pi_{2,2}^{*} = \left(\frac{1}{3}\right) \cdot (4{,}166.67 + 2777.78 + 277.78) = 2{,}407.41$$

$$\pi_{2,3}^{*} = \left(\frac{1}{3}\right) \cdot (3 \cdot 277.78) = 277.78$$

$$\pi_{1}^{*} = \left(\frac{1}{3}\right) \cdot (2777.78 + 2407.41 + 277.78) = 1{,}820.99$$

The expected value of the dual variable and the expected value of the optimal solution together with equation (16) are used to build Benders Cut to add to the master problem.

$$\begin{aligned} \alpha_{2,1} &\geq 72800 - 3703.70 \cdot (v_{2,1} - 0) \\ \alpha_{2,2} &\geq 58240 - 2407.41 \cdot (v_{2,2} - 6.05) \\ \alpha_{2,3} &\geq 28560 - 277.78 \cdot (v_{2,3} - 30.24) \end{aligned} \qquad (21)$$

$$\alpha_{1} \geq 36680 - 1820.99 \cdot (v_{1} - 36.29) \qquad (22)$$

Equation (21) shows the constraints that must be included in the sub-problems of the second stage. Similarly, equation (22) shows the constraint that must be included in the sub-problems of the first stage in the second iteration.

From Table 8 it can be observed that in stage 3, the natural inflows for sub -problems 1 and 4 (k_3= 1, 4) are not enough to meet recycle end volume conditions, so these sub problems are penalized with future costs. For these subproblems it can be observed that the thermal generators are generating at maximum capacity, the hydro gen is not generating and there are 10 MW of unserved energy. Therefore, the cost of these sub problems has two components, one based in the actual costs AC ($200) and the FCF estimation ($150). Both have to be multiplied for the stage duration, resulting in a total cost of $117,6000 ($67,200 + $50,400).

Convergence

The convergence is calculated in equation (25) using equation (23) for the Upper Bound, and equation (24) for the Lower Bound

$$Z_{upper} = \frac{1}{K} \sum_{k=1}^{K} AC_k \qquad (23)$$

$$Z_{lower} = AC_1 + FCF_1 \qquad (24)$$

$$\varepsilon = \frac{Z_{upper} - Z_{lower}}{Z_{upper}} \cdot 100 < \hat{\varepsilon} = 1\% \qquad (25)$$

Above equations take the following values:

$$Z_{upper} = 0 + \frac{(15120 + 2 \cdot 10080)}{3} + \frac{(3 \cdot 67200 + 33600 + 50400 + 2 \cdot 31920 + 28560 + 25200)}{9}$$
$$= 56{,}560$$

$$Z_{lower} = 0 + 0 = 0$$

$$\varepsilon = \frac{104{,}760 - 0}{104{,}760} \cdot 100 = 100\% > 1\%$$

ITERATION 2

Direction: Forward Pass

Equations (21) and (22) are added to the sub-problems of iteration 2 on the forward pass.
For stage 1:

$$Z_1(0) = min[G\_th_1 + 10USE_1 + \alpha_1]$$
$$G\_th_1 + G\_h_1 + USE_1 = 90$$
$$v_1 = 90.72 - 0.6048 \cdot G\_h_1$$
$$\alpha_1 \geq 0$$
$$\alpha_1 \geq 36680 - 1820.99 \cdot (v_1 - 36.29)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_1 \\ G\_th_1 \\ USE_1 \\ v_1 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 90 \\ 100 \end{bmatrix}$$

For stage 2:

$$Z_{2,1}(v_{t-1}^*) = min\left[G\_th_{2,1} + 10USE_{2,1} + \alpha_{2,1}\right]$$
$$G\_th_{2,1} + G\_h_{2,1} + USE_{2,1} = 160$$
$$v_{2,1} = v_{1,1} + u \cdot A_{2,1} - 0.6048 \cdot G\_h_{2,1}$$
$$\alpha_{2,1} \geq 0$$
$$\alpha_{2,1} \geq 72800 - 3703.70 \cdot \left(v_{2,1} - 0\right)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,1} \\ G\_th_{2,1} \\ USE_2 \\ v_2 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

$$Z_{2,2}(v_{t-1}^*) = min\left[G\_th_{2,2} + 10USE_{2,2} + \alpha_{2,2}\right]$$
$$G\_th_{2,2} + G\_h_{2,2} + USE_{2,2} = 160$$
$$v_{2,2} = v_{1,1} + u \cdot A_{2,2} - 0.6048 \cdot G\_h_{2,2}$$
$$\alpha_{2,2} \geq 0$$
$$\alpha_{2,2} \geq 58240 - 2407.41 \cdot \left(v_{2,2} - 6.05\right)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,2} \\ G\_th_{2,2} \\ USE_2 \\ v_2 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

$$Z_{2,3}(v_{t-1}^*) = min\left[G\_th_{2,3} + 10USE_{2,3} + \alpha_{2,3}\right]$$
$$G\_th_{2,3} + G\_h_{2,3} + USE_{2,3} = 160$$
$$v_{2,3} = v_{1,1} + u \cdot A_{2,3} - 0.6048 \cdot G\_h_{2,3}$$
$$\alpha_{2,3} \geq 0$$
$$\alpha_{2,3} \geq 28560 - 277.78 \cdot \left(v_{2,3} - 30.24\right)$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_h_{2,3}\\G\_th_{2,3}\\USE_3\\v_3\end{bmatrix} \leq \begin{bmatrix}100\\100\\160\\100\end{bmatrix}$$

We have now an approximation for the FCF, the stored volume at the end of stages 1 and 2 have a value different from 0 (see Figure 30).

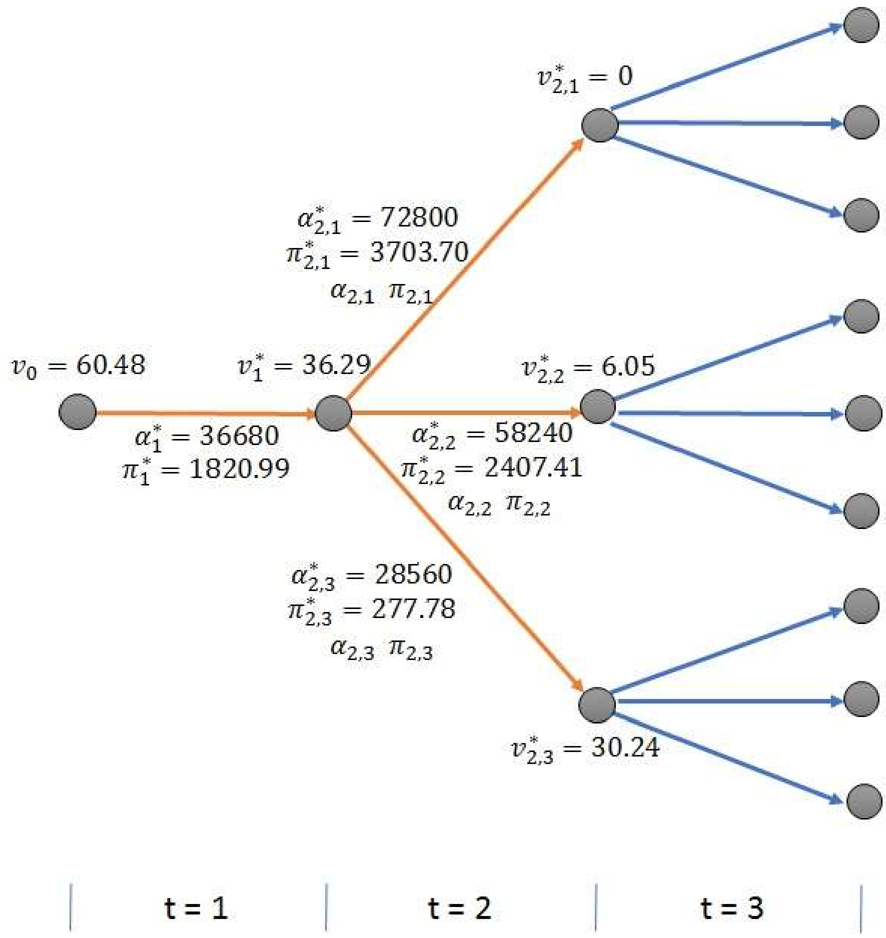


Figure 30. Iteration 2 Forward Pass.

Table 9 shows the results of iteration 2 in the forward pass direction.

Table 9: Results of Iteration 2 Forwards Pass

t	k	v_t^*	AC	FCF
1	1	56.43	5595.25	0
2	1	19.66	14984.75	0
	2	30.24	11204.75	0
	3	74.57	16800	16244.75

Direction: Backward Pass

Table 10 shows the results of iteration 2 in the backwards pass direction. This information can be used to generate new approximations.

Table 10: Results of Iteration 2 Backward Pass.

t	k	AC	FCF	$\pi_{t,k}$

3	1	46200	0	2777.78
	2	31500	0	277.78
	3	28140	0	277.78
	4	31920	0	277.78
	5	28560	0	277.78
	6	25200	0	277.78
	7	19605	0	277.78
	8	16245	0	277.78
	9	12885	0	277.78
2	1	16800	28018.98	111.11
	2	10080	29684.75	277.78
	3	16800	16244.75	277.78

Using the results in Table 10, three new cuts are calculated for the second stage (equation (26)) and one more for the first stage (equation (27)).

$$\alpha_{2,1} \geq 35280 - 111.11 \cdot \left(v_{2,1} - 19.66\right) \quad (26)$$
$$\alpha_{2,2} \geq 28560 - 277.78 \cdot \left(v_{2,2} - 30.24\right)$$
$$\alpha_{2,3} \geq 16244.75 - 277.78 \cdot \left(v_{2,3} - 74.57\right)$$
$$\alpha_1 \geq 39209.49 - 555.56 \cdot (v_1 - 56.43) \quad (27)$$

Convergence

The upper and lower bounds are higher than the desired gap as indicated below.

$$Z_{upper} = 46620$$
$$Z_{lower} = 5595.25$$
$$\varepsilon = 88\% > 1\%$$

ITERATION 3

Direction: Forward Pass

Equations (26) and (27) are added to the sub-problems of the forward pass of iteration 3.

For stage 1:

$$Z_1(0) = min[G\_th_1 + 10USE_1 + \alpha_1]$$
$$G\_th_1 + G\_h_1 + USE_1 = 90$$
$$v_1 = 30.24 - 0.6048 \cdot G\_h_1$$
$$\alpha_1 \geq 0$$
$$\alpha_1 \geq 36680 - 1820.99 \cdot (v_1 - 36.29)$$
$$\alpha_1 \geq 39209.49 - 555.56 \cdot (v_1 - 56.43)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_1 \\ G\_th_1 \\ USE_1 \\ v_1 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 90 \\ 100 \end{bmatrix}$$

For stage 2:

$$Z_{2,1}(v_{t-1}^*) = min\left[G\_th_{2,1} + 10USE_{2,1} + \alpha_{2,1}\right]$$
$$G\_th_{2,1} + G\_h_{2,1} + USE_{2,1} = 160$$
$$v_{2,1} = v_{1,1} + u \cdot A_{2,1} - 0.6048 \cdot G\_h_{2,1}$$
$$\alpha_{2,1} \geq 0$$
$$\alpha_{2,1} \geq 72800 - 3703.70 \cdot \left(v_{2,1} - 0\right)$$
$$\alpha_{2,1} \geq 35280 - 111.11 \cdot \left(v_{2,1} - 19.66\right)$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_h_{2,1}\\G\_th_{2,1}\\USE_2\\v_2\end{bmatrix} \leq \begin{bmatrix}100\\100\\160\\100\end{bmatrix}$$

$$Z_{2,2}(v_{t-1}^*) = min[G\_th_{2,2} + 10USE_{2,2} + \alpha_{2,2}]$$
$$G\_th_{2,2} + G\_h_{2,2} + USE_{2,2} = 160$$
$$v_{2,2} = v_{1,1} + u \cdot A_{2,2} - 0.6048 \cdot G\_h_{2,2}$$
$$\alpha_{2,2} \geq 0$$
$$\alpha_{2,2} \geq 58240 - 2407.41 \cdot (v_{2,2} - 6.05)$$
$$\alpha_{2,2} \geq 28560 - 277.78 \cdot (v_{2,2} - 30.24)$$
$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_h_{2,2}\\G\_th_{2,2}\\USE_2\\v_2\end{bmatrix} \leq \begin{bmatrix}100\\100\\160\\100\end{bmatrix}$$

$$Z_{2,3}(v_{t-1}^*) = min[G\_th_{2,3} + 10USE_{2,3} + \alpha_{2,3}]$$
$$G\_th_{2,3} + G\_h_{2,3} + USE_{2,3} = 160$$
$$v_{2,3} = v_{1,1} + u \cdot A_{2,3} - 0.6048 \cdot G\_h_{2,3}$$
$$\alpha_{2,3} \geq 0$$
$$\alpha_{2,3} \geq 28560 - 277.78 \cdot (v_{2,3} - 30.24)$$
$$\alpha_{2,3} \geq 16244.75 - 277.78 \cdot (v_{2,3} - 74.57)$$
$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_h_{2,3}\\G\_th_{2,3}\\USE_3\\v_3\end{bmatrix} \leq \begin{bmatrix}100\\100\\160\\100\end{bmatrix}$$

It can be observed that at the end of first stage, water has a higher opportunity cost in the future stages, so at the end of the first stage the solution stores a higher volume of water, as shown in Table 11.

Table 11: Results of Iteration 3 Forward Pass.

t	k	$v_{t,k}^*$	AC	FCF
1	1	90.72	15120	20160
2	1	36.29	10080	57120
	2	60.48	10080	36960
	3	84.67	10080	36960

Direction: Backward Pass

Table 12 shows the results of iteration 3 in the backward pass direction.

Table 12: Results of Iteration 3 Backward Pass.

t	k	AC	FCF	$\pi_{t,k}$
3	1	30240	0	277.78
	2	26880	0	277.78
	3	23520	0	277.78
	4	23520	0	277.78
	5	20160	0	277.78
	6	16800	0	277.78
	7	16800	0	277.78
	8	13440	0	277.78

	9	10080	0	277.78
2	1	10080	36960	0
	2	10080	20160	277.78
	3	14337.78	9182.22	277.78

There is information to calculate three more approximations for the second stage FCF (equation (28)) and one more for the first stage (equation (29)).

$$\alpha_{2,1} \geq 26880 - 277.78 \cdot (v_{2,1} - 36.29) \quad (28)$$
$$\alpha_{2,2} \geq 20160 - 277.78 \cdot (v_{2,2} - 60.48)$$
$$\alpha_{2,3} \geq 13440 - 277.78 \cdot (v_{2,3} - 84.67)$$
$$\alpha_1 \geq 33600 - 185.18 \cdot (v_1 - 90.72) \quad (29)$$

Convergence

The upper and lower bounds are higher than the desired gap as indicated below.

$$Z_{upper} = 45360$$
$$Z_{lower} = 35280$$
$$\varepsilon = 22.22\% > 1\%$$

ITERATION 4

Direction: Forward Pass

Equations (15) and (16) are added to the sub-problems of the forward pass of iteration 4.

For stage 1:

$$Z_1(0) = min[G\_th_1 + 10USE_1 + \alpha_1]$$
$$G\_th_1 + G\_h_1 + USE_1 = 90$$
$$v_1 = 30.24 - 0.6048 \cdot G\_h_1$$
$$\alpha_1 \geq 0$$
$$\alpha_1 \geq 36680 - 1820.99 \cdot (v_1 - 36.29)$$
$$\alpha_1 \geq 39209.49 - 555.56 \cdot (v_1 - 56.43)$$
$$\alpha_1 \geq 33600 - 185.18 \cdot (v_1 - 90.72)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_1 \\ G\_th_1 \\ USE_1 \\ v_1 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 90 \\ 100 \end{bmatrix}$$

For stage 2:

$$Z_{2,1}(v_{t-1}^*) = min[G\_th_{2,1} + 10USE_{2,1} + \alpha_{2,1}]$$
$$G\_th_{2,1} + G\_h_{2,1} + USE_{2,1} = 160$$
$$v_{2,1} = v_{1,1} + u \cdot A_{2,1} - 0.6048 \cdot G\_h_{2,1}$$
$$\alpha_{2,1} \geq 0$$
$$\alpha_{2,1} \geq 72800 - 3703.70 \cdot (v_{2,1} - 0)$$
$$\alpha_{2,1} \geq 35280 - 111.11 \cdot (v_{2,1} - 19.66)$$
$$\alpha_{2,1} \geq 26880 - 277.78 \cdot (v_{2,1} - 36.29)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,1} \\ G\_th_{2,1} \\ USE_2 \\ v_2 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

$$Z_{2,2}(v_{t-1}^*) = min[G\_th_{2,2} + 10USE_{2,2} + \alpha_{2,2}]$$
$$G\_th_{2,2} + G\_h_{2,2} + USE_{2,2} = 160$$

$$v_{2,2} = v_{1,1} + u \cdot A_{2,2} - 0.6048 \cdot G\_h_{2,2}$$
$$\alpha_{2,2} \geq 0$$
$$\alpha_{2,2} \geq 58240 - 2407.41 \cdot (v_{2,2} - 6.05)$$
$$\alpha_{2,2} \geq 28560 - 277.78 \cdot (v_{2,2} - 30.24)$$
$$\alpha_{2,2} \geq 20160 - 277.78 \cdot (v_{2,2} - 60.48)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,2} \\ G\_th_{2,2} \\ USE_2 \\ v_2 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

$$Z_{2,3}(v_{t-1}^*) = min[G\_th_{2,3} + 10USE_{2,3} + \alpha_{2,3}]$$
$$G\_th_{2,3} + G\_h_{2,3} + USE_{2,3} = 160$$
$$v_{2,3} = v_{1,1} + u \cdot A_{2,3} - 0.6048 \cdot G\_h_{2,3}$$
$$\alpha_{2,3} \geq 0$$
$$\alpha_{2,3} \geq 28560 - 277.78 \cdot (v_{2,3} - 30.24)$$
$$\alpha_{2,3} \geq 16244.75 - 277.78 \cdot (v_{2,3} - 74.57)$$
$$\alpha_{2,3} \geq 13440 - 277.78 \cdot (v_{2,3} - 84.67)$$
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq x = \begin{bmatrix} G\_h_{2,3} \\ G\_th_{2,3} \\ USE_3 \\ v_3 \end{bmatrix} \leq \begin{bmatrix} 100 \\ 100 \\ 160 \\ 100 \end{bmatrix}$$

Table 13 shows the results of iteration 4 in the forward pass direction.

Table 13: Results of iteration 4 Forwards Pass.

t	k	$v_{t,k}^*$	AC	FCF
1	1	54.43	5040	40320
2	1	24.19	16800	30240
	2	24.19	10080	30240
	3	72.58	16800	16800

Direction: Backward Pass

Table 14 shows the results of iteration 4 in the backward pass direction.

Table 14: Results of iteration 3 Backward Pass.

t	k	AC	FCF	$\pi_{t,k}$
3	1	33600	0	277.78
	2	30240	0	277.78
	3	26880	0	277.78
	4	33600	0	277.78
	5	30240	0	277.78
	6	26880	0	277.78
	7	20160	0	277.78
	8	16800	0	277.78
	9	13440	0	277.78
2	1	16800	30240	1111.11
	2	10080	30240	277.78
	3	16800	16800	277.78

Convergence

The upper and lower bounds are identical as indicated below, so the process is concluded and this iteration results are the optimal.

$$Z_{upper} = 45360$$
$$Z_{lower} = 45360$$
$$\varepsilon = 0 < 1\%$$

The expected cost of the problem is $45,360. Figure 31 show the optimal end volumes for each sub-problem.

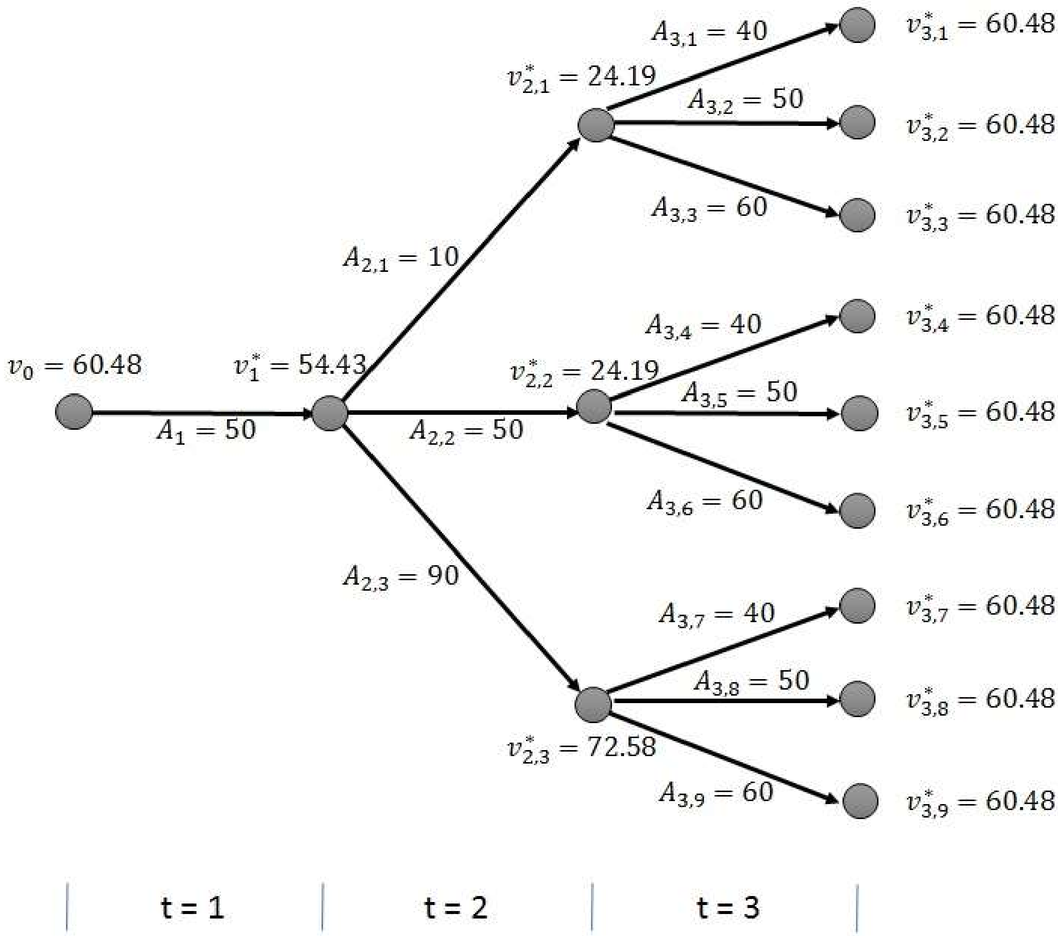


Figure 31. Optimum volumes per sub-problem.

Model solution using Hanging Branches

The equivalent tree using hanging branches method is showed below:

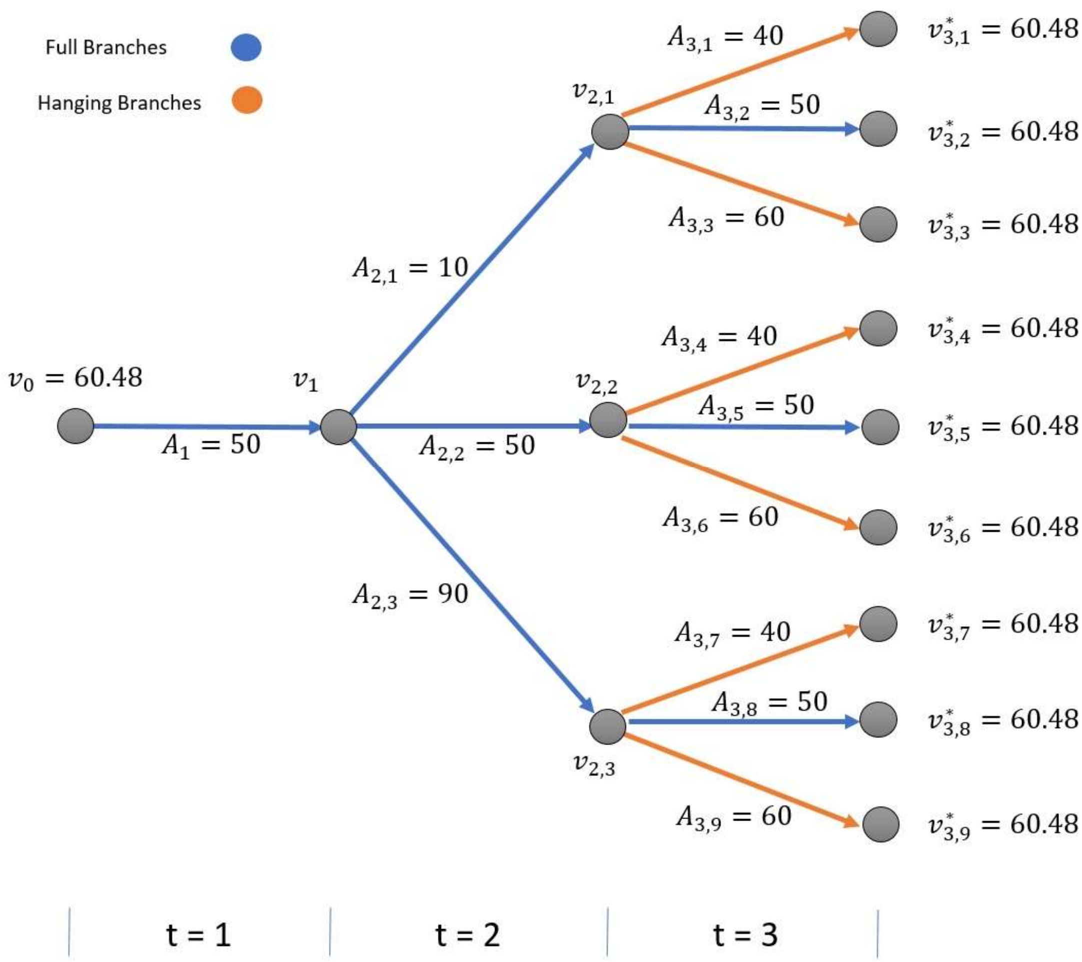


Figure 32: Multi-stage tree with full and hanging branches representation

The problem formulated as hanging branches formulation is the scenario – wise decomposition + non anticipativity constraint version of equation 17. The formulation can be summarized as follows:

$$Z = \min\left[\frac{1}{9}\sum_{k=1}^{9}[G\_th_{1,k} + 10 \cdot USE_{1,k} + G\_th_{2,k} + 10 \cdot USE_{2,k} + G\_th_{3,k} + 10 \cdot USE_{3,k}]\right]$$

$$G\_h_{1,k} + G\_th_{1,k} + USE_{1,k} = 90$$

$$G\_h_{2,k} + G\_th_{2,k} + USE_{2,k} = 160$$

$$G\_h_{3,k} + G\_th_{3,k} + USE_{3,k} = 110$$

$$v_{1,k} = v_{0,k} + 30.24 - 0.6048 \cdot G_{h_{1,k}}, \forall k = 1 \dots 9$$

$$v_{2,k} = v_{1,k} + 6.048 - 0.6048 \cdot G\_h_{2,k}, \forall k = 1 \dots 3$$

$$v_{2,k} = v_{1,k} + 30.24 - 0.6048 \cdot G\_h_{2,k}, \forall k = 4 \dots 6$$

$$v_{2,k} = v_{1,k} + 54.432 - 0.6048 \cdot G\_h_{2,k}, \forall k = 7 \dots 9$$

$$v_{3,k} = v_{2,k} + 48.384 - 1.2096 \cdot G\_h_{2,k}, \forall k = 1, 4, 7$$

$$v_{3,k} = v_{2,k} + 60.48 - 1.2096 \cdot G\_h_{2,k}, \forall k = 2, 5, 8$$

$$v_{3,k} = v_{2,k} + 72.576 - 1.2096 \cdot G\_h_{2,k}, \forall k = 3, 6, 9$$

$$\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix} \leq x = \begin{bmatrix}G\_th_{t,k}\\G\_h_{t,k}\\USE_{1,k}\\USE_{2,k}\\USE_{3,k}\\v_{t,k}\end{bmatrix} \leq \begin{bmatrix}100\\100\\90\\160\\110\\100\end{bmatrix}$$

$$v_{1,1} = v_{1,2} = v_{1,3} = \cdots = v_{1,9}$$

$$v_{2,1} = v_{2,2} = v_{2,3}$$

$$v_{2,4} = v_{2,5} = v_{2,6}$$

$$v_{2,7} = v_{2,8} = v_{2,9}$$

To represent the example in PLEXOS is needed the following objects:

- 2 generators
- 1 storage
- 1 region
- 1 variable
- 1 global

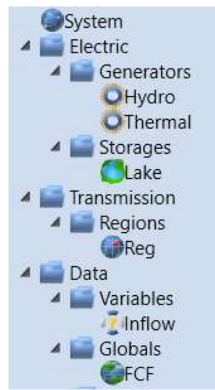
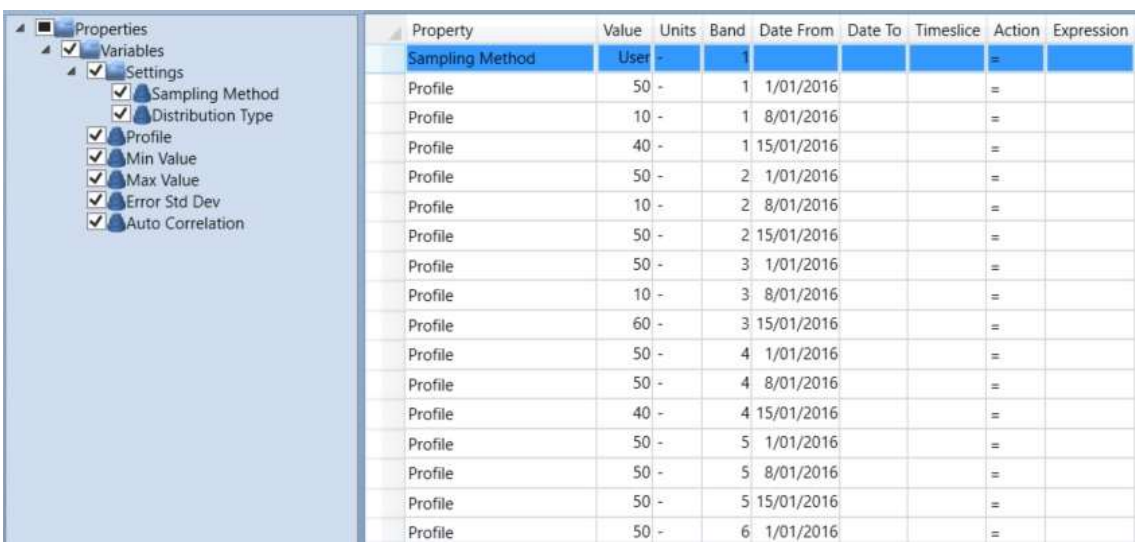


Figure 33: Objects tree representing the example



Property	Value	Units	Band	Date From	Date To	Timeslice	Action	Expression
Sampling Method	User	-	1				=	
Profile	50	-	1	1/01/2016			=	
Profile	10	-	1	8/01/2016			=	
Profile	40	-	1	15/01/2016			=	
Profile	50	-	2	1/01/2016			=	
Profile	10	-	2	8/01/2016			=	
Profile	50	-	2	15/01/2016			=	
Profile	50	-	3	1/01/2016			=	
Profile	10	-	3	8/01/2016			=	
Profile	60	-	3	15/01/2016			=	
Profile	50	-	4	1/01/2016			=	
Profile	50	-	4	8/01/2016			=	
Profile	40	-	4	15/01/2016			=	
Profile	50	-	5	1/01/2016			=	
Profile	50	-	5	8/01/2016			=	
Profile	50	-	5	15/01/2016			=	
Profile	50	-	6	1/01/2016			=	

Figure 34: Variable profile to represent uncertainty in PLEXOS

The global class has to be configured in the following way to have the tree modelled as in Figure 32.

Property	Value	Data File	Units	Band	Date From	Date To	Timeslice	Action	Expression	Scenario	Memo
Tree Stage Count	3		-	1				=			
Tree Period Type	interval		-	1				=			
Tree Stages Position	168		-	1				=			
Tree Stages Position	336		-	2				=			
Tree Stages Leaves	3		-	1				=			
Tree Stages Leaves	1		-	2				=			
Tree Stages Hanging Branches	0		-	1				=			
Tree Stages Hanging Branches	2		-	2				=			

Figure 35: Global class configuration

Once the problem is solved in MT, the objective function is 45,360 which is the same objective function value found using SDDP method in section 6.2.

```
MT Schedule Completed. Time: 00:00:02.1
------------------------------------------------------------------------------------------------------------------------------------
Minimization Objective Function:
    Linear Relaxation:..........................................................................          4.5360000000e+004
    Infeasibilities:............................................................................                          0
```

Figure 36: Objective function value in log file