Electrical Network Reduction in PLEXOS

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1 Background

For the sake of completeness we begin with the general case of steady-state AC power flow, wherein the relationship between current (I), admittance (Y), and voltage (V) is:

$$I = YV \tag{1}$$

All three quantities defined above are complex-valued. Also, for an N-node transmission network, I and V are Nx1 column vectors and Y is an NxN matrix.

Defining i as the set of "internal" nodes, i.e., those of interest to the analyst, and e as the set of "external" nodes, equation 1 can be partitioned as follows:

$$\begin{bmatrix} I_i \\ I_e \end{bmatrix} = \begin{bmatrix} Y_{ii} & Y_{ie} \\ Y_{ei} & Y_{ee} \end{bmatrix} \begin{bmatrix} V_i \\ V_e \end{bmatrix} \tag{2}$$

With the goal of eliminating the external network, some basic algebraic manipulation of equation 2 is in order:

$$I_i = Y_{ii}V_i + Y_{ie}V_e \tag{3a}$$
$$I_e = Y_{ei}V_i + Y_{ee}V_e \tag{3b}$$

From equation 3b we can express V_e in terms of I_e and V_i:

$$V_e = (I_e - Y_{ei}V_i)Y_{ee}^{-1} \tag{4}$$

Substituting the above expression for V_e into equation 3a and rearranging terms we get:

$$I_i - Y_{ie}Y_{ee}^{-1}I_e = (Y_{ii} - Y_{ie}Y_{ee}^{-1}Y_{ei})V_i \tag{5}$$

The above equation is the basis of the algorithm for eliminating the external network from the original and replacing it with an electrical-equivalent at the boundary. To accomplish this, a physical understanding of the mathematical terms in equation 5 is necessary.

We note first that the expression $(Y_{ii} - Y_{ie}Y_{ee}^{-1}Y_{ei})$ on the right-hand side is the Schur complement of the Y_{ee} block of Y. The term $-Y_{ie}Y_{ee}^{-1}Y_{ei}$, denoted by $\tilde{Y}_{ii}$ hereafter, encapsulates the admittances of ties between internal and external nodes. In order to incorporate these admittances into a reduced (internal) network that is externally aware, virtual lines connecting boundary nodes need to be created programmatically, as indicated by nonzero elements of $\tilde{Y}_{ii}$.

The term $-Y_{ie}Y_{ee}^{-1}I_e$, denoted by $\tilde{I}_i$ hereafter, is comprised of current transactions at boundary nodes. For the reduction to be exact in terms of internal voltages and currents from the original network, the current transactions need to be fixed. However, it is more convenient to deal with power transactions, so further manipulation of equation 5 is necessary. We first rewrite it in an abbreviated form as follows:

$$I_i + \tilde{I}_i = (Y_{ii} + \tilde{Y}_{ii})V_i \tag{6}$$

Upon taking the complex conjugate (denoted by $*$) of both sides of the above equation and then multiplying by V_i we get:

$$V_i I_i^* + V_i \tilde{I}_i^* = V_i (Y_{ii}^* + \tilde{Y}_{ii}^*) V_i^* \tag{7}$$

We finally have equation 5 in a form that is suitable to capture external-equivalent power transactions. Specifically, nonzero elements of the $V_i \tilde{I}_i^*$ vector correspond to virtual generators and/or loads at the boundary that need to be created programmatically.

In summary, to carve out a reduced (internal) electrical network in a manner that properly accounts for power flows in and out at the interface with the external network, the two-step procedure is:

1. Create virtual lines corresponding to nonzero elements of $\tilde{Y}_{ii}$.
2. Create virtual generators/loads corresponding to nonzero elements of $V_i \tilde{I}_i^*$.

2 Implementation

Consider the following linearized version of equation 1:

$$P = B\theta \tag{8}$$

P is the net power injection vector; B, the susceptance matrix, i.e., $-\Im(Y)$ with shunt elements neglected; and θ, the voltage angle (radians) vector. The dimensions of P, B, and θ are Nx1, NxN, and Nx1, respectively.

Upon following an identical procedure to that employed in equations 2-4 we get:

$$P_i - B_{ie}B_{ee}^{-1}P_e = (B_{ii} - B_{ie}B_{ee}^{-1}B_{ei})\theta_i \tag{9}$$

As detailed in §1, to carve out a reduced electrical network the two-step procedure is:

1. Create virtual lines corresponding to nonzero elements of $-B_{ie}B_{ee}^{-1}B_{ei}$.
2. Create virtual generators/loads corresponding to nonzero elements of $-B_{ie}B_{ee}^{-1}P_e$.

In PLEXOS, these virtual elements are created before phase execution; however, they are activated at the beginning of the ST phase. Specifically, virtual generators/loads are implemented using market objects. For each interval, the MT solution is used to estimate both P_e and the price paid (received) for the sales (purchases) to (from) these virtual markets. It is worth emphasizing that, as formulated, there is no obligation for the internal network to meet (use) the estimated external demand (supply).