Game Theory Models in PLEXOS

Game Theory Models of Imperfect Competition in the PLEXOS Software

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I: Background

At the most basic level, competitive electricity markets result from the efficient trade of two goods - energy and transmission services. Models of imperfect competition utilise game theory to examine behavioural deviations by market participants from the competitive norm and their impacts on efficient trade in these markets. This behaviour, however, is not independent from market design and structure, which affect participant incentives and interactions. For example, auction design, *e.g.* single- versus multi-part bids, in a centralised market may give participants certain opportunities to exercise market power. Likewise, institutional arrangements in a bilateral market may give arbitragers incentives to act in a non-competitive manner. Consequently, if market design or institutional arrangements are poor then inefficiencies may be created that erode the potential benefits from trade.

As a result, one way to categorise models of power markets is by the market mechanism:

- centralised – POOLCO Model
- decentralised – Bilateral Model

The majority of market power studies to date either explicitly or implicitly assume a centralised auction process, administered by an Independent System Operator (ISO), through which generators sell energy to consumers [3,7,11]. A growing number of studies typically assume a decentralised trading process by which generators sell to consumers bilaterally through power exchanges or arbitragers [14]. Metzler *et al.* [10] presents models that can represent both POOLCO and Bilateral Models.

A second way to group these models is by the type of interaction that they assume about the behaviour of participants (primarily, but not limited to, generators):

- Competitive – firms are price-takers and possess no market power

- Cournot – quantity is the strategic variable, and firms choose quantities simultaneously, under the assumption that other firms' quantities are fixed
- Bertrand – price is the strategic variable, and firms choose prices simultaneously, assuming that other firms' prices are fixed
- Supply Function Equilibrium (SFE) – entire bid functions are the strategic variables, and firms choose their supply functions simultaneously, under the assumption that other firms' supply functions are fixed; a market mechanism, *e.g.* an ISO, then determines price and sets the quantity.

These are the basic paradigms currently relevant to this article. Other paradigms include, but are not restricted to, Stackelberg behaviour and collusive behaviour. These paradigms are presently outside the scope of this article.

The application of the different paradigms of participant interaction may produce equilibrium outcomes with substantial variations in the intensity of competition. Consequently, equilibrium outcomes may be sensitive to the particular paradigm selected. This sensitivity has created significant discussion regarding the relevance of each paradigm to modelling competition in electricity markets and has led to the development of a number of variants to each paradigm that investigate different assumptions about market structure and design.

Section II first explains the two primary market mechanisms relevant to modelling electricity markets, and Section III contains a detailed discussion of the specific models used in PLEXOS.

II: Market Mechanisms

True competition in electricity markets, in which competing generators sell directly to unaffiliated distributors / retailers, did not emerge until the 1990s. Prior to this time, real markets did not exist, and isolated 'pockets' of generation units operated as localised monopolies. With the passage of time, the gradual growth in transmission network interconnection facilitated bilateral contracting between local monopolies – a seller would contract with a buyer to deliver a certain quantity of electricity at a specific time, and the buyer would factor this input into its unpriced, centralised decision-making process. With scattered monopolies using networks more or less exclusively, administrative rules were sufficient to manage externalities and to coordinate security of supply. The emergence of political pressures for competition in electricity led to demands for network access and resulted in increasingly complex network externalities.

The emergence of real markets for electricity occurred with the use of locational pricing to internalise complex network externalities and with the implementation of the concept of the Independent System Operator (ISO) to operate a centralised spot market synchronised with real-time, physical system operations. The fact that electricity is non-storable and supply must satisfy demand in real time imposes a requirement for centralised system control. From an economic perspective, the most efficient way for the ISO to meet this objective is through the 'economic dispatch' – minimisation of the total cost of system operation given network and security constraints. The dual variables of the economic dispatch yield the spot prices that vary with time and with network location if constraints exist. These dual variables play a critical role in communicating spot price information to market participants - producers and consumers.

The theory of spot prices defines two distinct commodities in an electricity market – energy and transmission services – and the competitive operation of an electricity market depends on the efficient trade of both of these commodities and the existence of well-designed institutions to facilitate and support such trade [9]. Over the last decade, intense debate has emerged worldwide regarding the appropriate mechanism for the organisation of trade in electricity markets and the method for communicating these dual prices to market participants. The centralised mechanism is the POOLCO Model, in which the tasks of determining the economic dispatch and ensuring system security are the domain of a regulated ISO. The decentralized mechanism is the Bilateral Model, which relies on market mechanisms to achieve an economic dispatch, while relegating the task of system security to the ISO. It is possible that the ISO owns the transmission assets (TRANSCO Model), as long as it remains unaffiliated with any market participants. This distinction, however, is irrelevant for modelling purposes in this article. Although the definition of what specific institutional and economic features characterise each of the POOLCO and Bilateral models varies across policy circles, these descriptions characterise the models in a general context and *as they are used in this article.* It is beyond the present scope of this article to discuss the arguments for and against these two models.

In the POOLCO Model, the ISO uses a linear programming model to calculate the instantaneous spot price of both energy and transmission services (extremely similar to the economic dispatch), where the price of the transmission services reflects both transmission congestion and resistance losses. These instantaneous spot prices are often referred to in the literature as 'dispatch-based' prices [12]. From an economic perspective, the ISO mimics the role of a centralised agent that trades energy and transmission services with producers and consumers. If the transactions between the ISO and producers / consumers are valued at the optimal dispatch-based prices then the result is the perfectly competitive outcome in which all agents, including the ISO, act to maximize their profits (taking the prices as given). With efficient, *i.e.* location-based, prices for transmission services, variation in prices between two locations in the network represents the spot price of congestion between those locations. The use of location- or congestion-based pricing in conjunction with financial transmission rights (FTRs) between specific network locations enables market participants to hedge against volatile congestion prices [9,13].

The premise of the Bilateral Model is that decentralized market mechanisms can replicate the economic dispatch of a centralised spot market in the absence of strategic behaviour by participants. The role of the ISO is limited to providing transmission capacity and managing system security, or in some models, strictly the latter. In the Bilateral Model, market mechanisms determine the prices of energy and transmission services at pre-dispatch time, and trade between producers and consumers occurs through exchanges or arbitragers. The Bilateral Model deals with transmission services in one of two ways. The first approach is the definition of property rights on the transmission lines and the establishment of markets for these rights [4,5]. The second approach assumes that the arbitragers include requests for injections and withdrawals from the network by participants in their multi-nodal transactions [17].

These two models represent centralised versus decentralised approaches to the organisation of trade in an electricity market. The models produce equivalent economic outcomes with full information among market participants, completely defined property rights to energy and transmission services, the absence of market power among participants, and given appropriate and effective institutional structures to support the trading arrangements (see Boucher and Smeers [ ]). The trend in real world electricity markets is to utilise some form of hybrid of these extremes, such that an ISO manages a centralised auction that is complemented by bilateral markets in which the majority of trade takes place.

III: The Models

The models discussed in this article are variants of these general paradigms:

- Cournot;
- Bertrand; and

- Supply Function Equilibrium.

A general overview of these paradigms is in the Drayton Analytics Research Paper *Game Theory and Electricity Markets*.

IIIA: The Cournot Model: Overview

The purpose of this section is to give a general idea of the ‘common’ components that are typically part of Cournot models and to identify some modelling issues that are relevant when applying the Cournot model to electricity markets. It is not intended to be a comprehensive guide or literature review on this topic. Daxhelet and Smeers [ ] provide an overview of market modelling approaches.

IIIA.1: Generator Behaviour

In a typical Cournot model of imperfect competition among generators in an electricity market, an individual generator chooses its output level to maximize its profit, with the assumption that its rivals keep their output levels fixed. Similar assumptions are that producers choose a level of sales to a particular region given that competitors’ sales to the region are fixed. Beyond the basic assumption that defines the nature of Cournot competition, these models must also explicitly or implicitly address several other aspects of generator behaviour, in particular, generator assumptions regarding i) transmission prices and ii) the behaviour of arbitragers.

In some Cournot models, producers recognise the effect of their output decisions on transmission constraints and correctly anticipate the impact on transmission prices [3,11]. Such models, however, are not numerically solvable for large electricity systems. As a result, other models use the alternative assumption that generators naively ignore the impact of their output decisions on transmission prices. Although this assumption compromises realism in certain situations, it is critical to enable the solution of Cournot models for large electricity networks. This latter assumption is a central component of the models presented in section 3.4.

Cournot models with arbitrage require an assumption about generators’ expectations of arbitragers’ decisions. The first possibility is that generators expect that the arbitragers will adjust their purchases and sales in response to changes in their output decisions. This assumption is equivalent to a Stackelberg conjecture regarding arbitrager behaviour and results in ‘endogenous arbitrage’. The alternative possibility is that generators do not expect arbitragers to change their purchase and sales decisions at each node in response to generator output decisions. This assumption is equivalent to a Cournot conjecture regarding arbitrager behaviour and results in ‘exogenous arbitrage’.

IIIA.2: Transmission

A common approach to modelling the *physical* transmission network in the literature is the use of a linearized DC load flow model that is consistent with both Kirchhoff's Voltage Law and Kirchhoff's Current Law for real power flows [7,10,11,13]. By effectively modelling both of Kirchhoff's laws, this approach captures the important possibility that the nature of transmission enables generators to manipulate flows and constraints under certain conditions. Other studies completely ignore transmission constraints [6,16] or use transhipment models that disregard Kirchhoff's Voltage Law [1,8].

Models differ in the transmission pricing policy selected. Efficient transmission pricing models location–based, or spot, prices for transmission (or their equivalent), which capture the differences in transmission costs attributable to network congestion [13]. One way to model such a policy is to allow the ISO to charge a congestion-based fee for the transmission of electricity from an arbitrary hub node to a destination node. The ISO maximises the total value of transmission services by rationing scarce transmission capacity, based on the willingness to pay for these services. This approach is equivalent to modelling a competitive market for transmission services in which participants possess no market power.

It is also possible to model other transmission pricing policies, such as zonal pricing. With these alternative policies, however, it is also necessary to assume that the ISO uses some efficient, non-price mechanism to relieve congestion in order that network flows respect transmission capacity limits.

IIIA.3: Arbitrager Behaviour

Arbitragers are typically a component of the Bilateral Model; however, in a POOLCO Model with locational pricing, arbitrage is an implicit component of the model. Specifically, the ISO accepts supply and demand bids from generators and consumers (respectively) in order to maximize the net benefits of market participants. Modelling this approach implies that the ISO mimics the role of a 'perfect arbitrager' that purchases low cost power at one node and resells it at other nodes where the price is greater than the purchasing cost plus the transmission cost.

In a Bilateral Model, competitive arbitragers facilitate trade between generators and consumers by purchasing electricity at location i at price p_i and reselling it at location j at price p_j when $p_j > p_i + w_{ij}$ (where w_{ij} represents the transmission cost from node i to node j). The presence of competitive arbitragers in the power market eliminates all non-cost differences in prices across the network. Without arbitrage, Cournot competition by producers can create price differences across the network, even in the absence of transmission constraints.

A model by [14] assumes competitive behaviour among generators but imperfect competition among arbitragers. In the model, each arbitrager assumes that its rivals will not alter the quantities that they buy and sell (Cournot assumption).

IIIA.4: Consumers

Models of Cournot competition among generators typically assume that consumers act as price-takers in the power market.

IIIB: Overview of Selected Cournot Models in PLEXOS

This section relies significantly on material in [10]. The set of Cournot models that are the focus of this article (*Models I*, *IIa*, *IIb*) share the following common assumptions:

- producers behave as Cournot players, *i.e.* each individual producer chooses its output to maximize its profit given its rivals' outputs are fixed;
- the transmission system is represented as a linearized DC network on which power flows are consistent with Kirchhoff's laws;
- transmission pricing is based on locational marginal pricing; and
- both producers and arbitragers make output and pricing decisions under the assumption that their decisions do not affect transmission constraints and the fees that the ISO charges for transmission services.

Figure 1 illustrates how these models relate to each other, as well as to several POOLCO models.

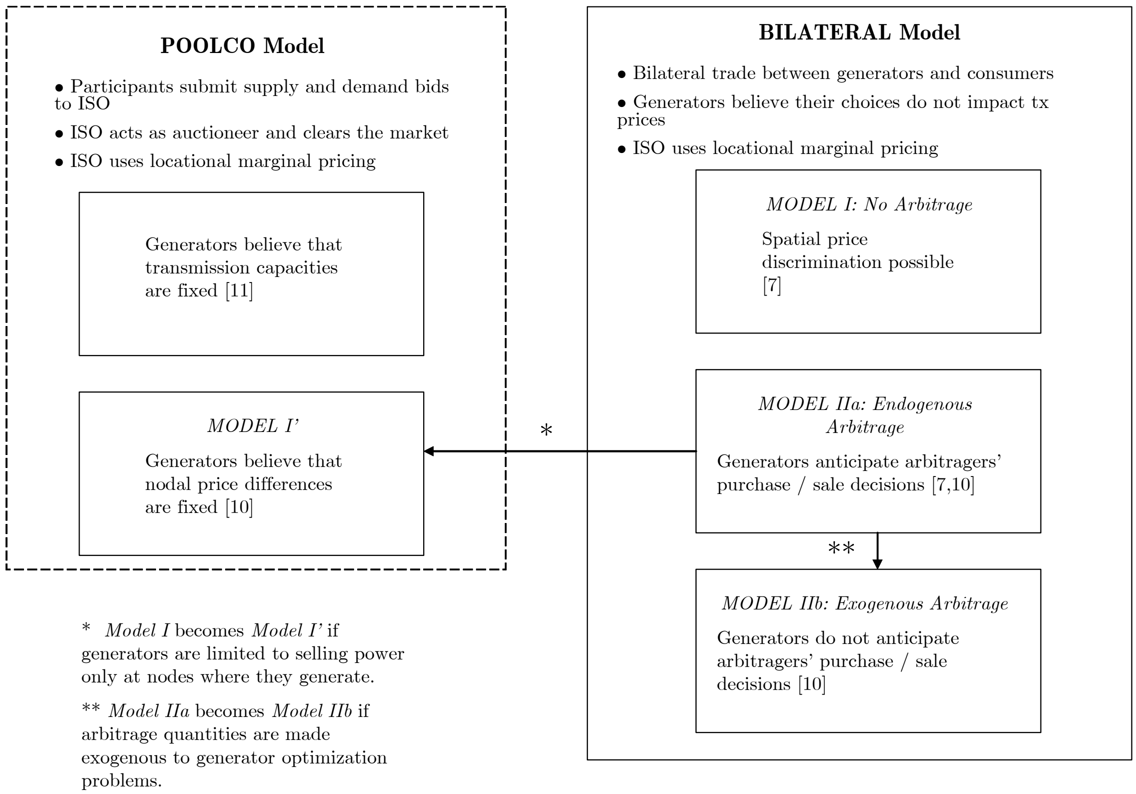

Figure **1**: *Selected Cournot Models*

Source: Adapted (and modified) from [10].

The Cournot models implemented in PLEXOS simulate imperfect competition among generators that produce and sell electricity in a Bilateral Model. It has been demonstrated that if perfect competition exists among arbitragers in a Bilateral Model, then the POOLCO Model and the Bilateral Model yield the same equilibrium prices under either perfect competition [2] or Cournot competition [10].

The first model (*Model I*) assumes that Cournot behaviour occurs without the possibility of arbitrage in the market. As a result, price differences arise in different locations around the network that are not cost-based. The second model (*Model IIb*) introduces the possibility of arbitrage, such that competitive arbitragers buy and sell electricity in the power market. The arbitragers' competitive behaviour eliminates any non-cost differences in prices across the network. *Model IIa* is not reviewed in this article because it is not implemented in PLEXOS for computational reasons. For a detailed description of *Model IIa*, see [7,10].

These models and their variants can be summarised as follows:

- No Arbitrage (*Model I*) – Cournot suppliers contract with customers, and no arbitragers exist in the market

- Arbitrage – Cournot suppliers contract with customers, and competitive arbitrage forces nodal price differences to equal the transmission cost
  - Stackelberg Conjecture – Endogenous Arbitrage (*Model IIa – not presented*) – suppliers expect that arbitragers will adjust their purchases / sales in the market in response to changes in supplier decisions
  - Cournot Conjecture – Exogenous Arbitrage (*Model IIb*) – suppliers make output and price decisions under the assumption that arbitragers will not change their purchases / sales at each node

IIIC: Common Modelling Aspects

Models I, *IIa*, *IIb* include common assumptions regarding: i) transmission, ii) producers, and iii) consumers. They are also characterised by the same form of solution. The existence of exogenous arbitrage is unique to *Model IIb*. This section provides a detailed description of each of the common model components.

IIIC.1: Transmission

IIIC.1.a: Description of the Network Model

The electricity network is modelled with a graph that uses a finite set of nodes (N) and a finite set of directed arcs (A). In general, constraints on transmission are represented by a finite number of linear inequalities. For purposes of the models, the constraints are limited to i) constraints on the transmission capacities of any of the arcs, or reverse arcs, in K, and to ii) constraints on flow balance.

The models use Kirchhoff's laws and Power Distribution Factors (*PDF*s) to implement a linearized DC network flow model [13]. A *PDF* represents the MW increase in flow resulting from 1MW of injection at an arbitrary hub, node H, and 1MW of power withdrawal at node i, due to Kirchhoff's laws. Under a linearized DC model, power is analogous to current, such that power flows satisfy analogies to Kirchhoff's Current Law and Kirchhoff's Voltage Law. For any physical network and an assumed set of network injections and withdrawals, these laws uniquely determine the flows – unlike a 'transshipment network', in which discretion generally exists in regard to the 'routing' of the network flows.

For example, consider a three-node (triangular) network in which each pair of nodes is connected directly by a line with an electrical reactance of 2.5. By Kirchhoff's laws, an injection of 10MW at node 3 and a 10MW withdrawal at node 1 will result in the following power flows:

- 3.33MW from node 3 to node 2;
- 3.33MW from node 2 to node 1; and
- 6.67MW from node 3 to node 1.

These flows satisfy Kirchhoff's Current Law since net flow into each node is zero, and they also satisfy Kirchhoff's Voltage Law since the net drop around any loop in the network is zero. In a linearized DC load flow model, the change in voltage angle over an arc is proportional to the product of the power flow and the reactance, *e.g.* proceeding around the voltage loop for the previous example, *i.e.* 3-2-1-3, gives a net voltage drop of 2.5 x (3.33) + 2.5 x (3.33) + 2.5 x (-6.67) = 0. An example of a *PDF* for a triangular network, with node 3 as the hub node, gives PDF_{ik} = -.33 for node $i = 1$ and the arc k in the direction from node 1 to node 2.

Since the Kirchhoff equations that determine flows in the DC model are linear, the principle of superposition applies; therefore, the flows induced by an injection of x at node i and a withdrawal of x at node j equal the sum of the flows induced by injecting x at node i and withdrawing x at the hub, node H, plus the flows induced by injecting x at node H and withdrawing x at node j. Due to the linearity of the DC network approximation, all generation is notionally routed through node H. The linearity property also implies that *the choice of the hub node is arbitrary*.

IIIC.1.b: Independent System Operator (ISO)

The ISO, by assumption, rations scarce interface capacity to maximize the value of transmission services that it provides to the market. Equivalently, the ISO maximizes its profit from selling transmission services (y_i) to the market given the transmission fees (w_i) that it charges to producers (and arbitragers in *Model IIb*) and given interface constraints.

For transmission services, $y_i > 0$ implies a net flow into node i while $y_i < 0$ implies a net flow from node i. The transmission fee is the cost that a producer (or arbitrager) pays the ISO for the (notional) transmission of power from node H to node i. This assumption is also equivalent to a competitive market for transmission rights. Further, the ISO maximizes its profit under the assumption that it cannot strategically manipulate the fees that it charges for transmission services.

In the models, the following notation is used for transmission:

- set of transmission nodes — N.
- set of transmission arcs — A.
- set of transmission interfaces — K.
- transmission capacity on arc k — T_k^+.
- transmission capacity on arc k in the reverse direction — T_k^-.
- transmission (MW) from the hub node (H) to node i — y_i.

- power distribution factor for node i on arc k, representing the MW increase in flow resulting from 1MW of injection at node H and 1 MW of power withdrawal at node i, due to Kirchhoff's laws. PDF_{ik}.
- transmission fee for transport from node H to i w_i.
- price at the hub, node H p_H.

IIIC.2: Consumers

Consumers at node i consume q_i MW such that the price at node i (p_i) is related linearly to consumption by the nodal demand function:

(1) $p_i(q_i) = P_i^0 - (P_i^0 \,/\, Q_i^0)q_i$.

In the demand function, P_i^0 is the vertical intercept, and Q_i^0 is the horizontal intercept. The use of a linear demand function allows for the use of a Linear Complementarity or Quadratic Problem (LCP or QP) formulation in the model. The use of a non-linear demand function would result in a Non-linear Complementarity Problem (NCP), which is, in general, more difficult to solve.

IIIC.3: Producers

Each individual producer $f : f \in F$ is located at a node. A producer has a generation capacity (MW) of G_{if}, and its generation (MWh) at node i is represented by g_{if}.

A producer maximizes its profit, *i.e.* revenues less costs, under an assumption of imperfect competition. The imperfect competition is modelled as Cournot behaviour with respect to sales in the power market. A producer has the option to sell its generation at any node; therefore, the sales at node i by producer f are s_{if}. Producer revenue is the product of sales at a node and the nodal price, summed across all nodes at which sales occur. The production cost possesses two components: producer marginal production cost (C_{if}) and the transmission fee (w_i). The constraints on the producer are i) generation station capacity, ii) production and sales balance, and iii) nonnegativity of both production and sales.

A critical assumption of the models is that producers do *not* anticipate the impact of their output decisions on transmission congestion and prices. This assumption, therefore, eliminates the situations in which producers manipulate transmission in a strategic manner to their benefit. This assumption diverges from the assumptions of the models in [3,11], in which producers anticipate (correctly) the effect of their decisions on transmission limits and on the resulting transmission prices.

The disadvantage of the assumption is that it does not allow for the possibility that, in reality, producers recognise the impact of their decisions on transmission prices. The advantage, however, is that the models are solvable for realistically large networks. The primary reason for this improvement in solvability is that the representation of producers' expectations - of their output decisions on transmission prices - requires that each producer's optimization problem includes the Karush-Kuhn-Tucker (KKT) conditions of the ISO's optimal power flow problem. The resulting mathematical program is highly non-convex and may possess multiple, local optima. Consequently, this simplifying assumption represents a tradeoff between a possibly more realistic representation of producer behaviour and computability.

IIIC.4: Solution

Given the assumptions on market design and market participant interaction, each model determines a market equilibrium. An equilibrium is defined as a set of generator outputs, consumer demands, prices, transmission flows, and arbitrage quantities (Model *IIb*) that satisfy the market participants' profit maximization conditions and market-clearing condition. A solution to each model that satisfies these conditions possesses the Nash equilibrium property – no participant has the incentive to unilaterally deviate from its equilibrium choices.

IIID: The Models

This section contains a detailed description of the Cournot models implemented in PLEXOS.

IIID.1: Cournot Model I: No Arbitrage

Model I is derived from [7]. An equivalent model is developed by [15] and includes the possibility of generation capacity expansion. This model proves existence and uniqueness of a solution by variational inequality methods. In addition, [10] proves existence and uniqueness using complementarity methods.

IIID.1.a: Description and Model-specific Assumptions

In *Model I*, power flows on a linearized DC network and the ISO allocates scarce transmission capacity efficiently. Trade in the electricity market occurs consistent with the Bilateral Model, such that producers, *i.e.* generators, contract directly with consumers for power supply. No arbitragers exist in the market. Since no arbitrage occurs between locations in the network, the Cournot assumption on generator behaviour enables non-cost based price differences to arise in different locations across the network.

In summary, the following assumptions on producer, consumer, and transmission behaviour characterise *Model I*:

- power flows over a linearized DC network;
- oligopolistic producers behave as Cournot suppliers in a Bilateral Model;
- transmission capacity is allocated efficiently; and
- arbitragers do not exist in the power market to eliminate non-cost price differences.

In this and subsequent formulations, the *superscript* on the profit function ( ) references the specific model. The LaGrange multipliers appear in parentheses.

IIID.1.b: Linear Complementarity Problem (LCP)

The collection of conditions that define the LCP solution to *Model I* originate from: i) the producer problem, ii) the ISO problem, and the iii) market-clearing equations.

Producer Problem

An individual producer, $\bar{f} \in F$, chooses generation ($g_{i\bar{f}}, \forall i$) and sales ($s_{i\bar{f}}, \forall i$) to maximize its profit ($\Pi_{\bar{f}}^{I}$)

(*2*) $$\Pi_{\bar{f}}^{I} = \sum_i \left[P_i^0 - \frac{P_i^0}{Q_i^0} \left(\sum_f s_{if} \right) - w_i \right] s_{i\bar{f}} - \sum_i \left(C_{i\bar{f}} - w_i \right) g_{i\bar{f}}$$

subject to the constraints

(*2a*) $g_{i\bar{f}} \le G_{i\bar{f}}, \forall i \in N$ $\qquad$ ($\gamma_{i\bar{f}}$)

(*2b*) $\sum_i s_{i\bar{f}} = \sum_i g_{i\bar{f}}$ $\qquad$ ($\alpha_{\bar{f}}$)

(*2c*) $s_{i\bar{f}} \ge 0, g_{i\bar{f}} \ge 0, \forall i \in N$.

The KKT conditions for [$\Pi_{\bar{f}}^{I}$] are:

- $s_{i\bar{f}}, \forall i$:

(*3a*) $$\left[\left(P_i^0 - \frac{P_i^0}{Q_i^0} \left(2s_{i\bar{f}} + \sum_{f \ne \bar{f}} s_{if} \right) \right) - w_i \right] - \alpha_{\bar{f}} \le 0$$

(*3b*) $s_{i\bar{f}} \ge 0$ $$s_{i\bar{f}} \left\{ \left[\left(P_i^0 - \frac{P_i^0}{Q_i^0} \left(2s_{i\bar{f}} + \sum_{f \ne \bar{f}} s_{if} \right) \right) - w_i \right] - \alpha_{\bar{f}} \right\} = 0$$

- $g_{i\bar{f}}, \forall i$:

(*4a*) $-\left(C_{i\bar{f}} - w_i \right) - \gamma_{i\bar{f}} + \alpha_{\bar{f}} \le 0$

(*4b*) $g_{i\bar{f}} \ge 0$ $\qquad$ $g_{i\bar{f}} \left\{ -\left(C_{i\bar{f}} - w_i \right) - \gamma_{i\bar{f}} + \alpha_{\bar{f}} \right\} = 0$

- $\gamma_{i\bar{f}}, \forall i$:

(*5a*) $g_{i\bar{f}} \le G_{i\bar{f}}$

(*5b*) $\gamma_{i\bar{f}} \ge 0$ $\qquad$ $\gamma_{i\bar{f}} \left(g_{i\bar{f}} - G_{i\bar{f}} \right) = 0$

- $\alpha_{\bar{f}}$:

(*6*) $\sum_i s_{i\bar{f}} = \sum_i g_{i\bar{f}}$.

ISO Problem

Given the transmission fees, the ISO chooses the provision of transmission services (y_i) to maximize its profit (Π^I_{ISO}), under the assumption that it cannot manipulate the fees that it receives for these services:

(*7*) $$\Pi^I_{ISO} = \sum_i w_i y_i \ ,$$

subject to the constraints

(*8a*) $$\sum_i PDF_{ik} y_i \le T_k^+, \forall k \in A \qquad (\lambda_k^+)$$

(*8b*) $$-\sum_i PDF_{ik} y_i \le T_k^-, \forall k \in A \qquad (\lambda_k^-)$$

The KKT conditions are:

- $y_i, \forall i$:

(*9*) $$w_i + \sum_i PDF_{ik} \left[\lambda_k^- - \lambda_k^+ \right] = 0$$

- $\lambda_k^+, \forall k$:

(*10a*) $$\sum_i PDF_{ik} y_i - T_k^+ \le 0$$

(*10b*) $$\lambda_k^+ \ge 0 \qquad \lambda_k^+ \left\{ T_k^+ - \sum_i PDF_{ik} y_i \right\} = 0$$

- $\lambda_k^-, \forall k$:

(*11a*) $$-\sum_i PDF_{ik} y_i - T_k^- \le 0$$

(*11b*) $$\lambda_k^- \ge 0 \qquad \lambda_k^- \left\{ T_k^- + \sum_i PDF_{ik} y_i \right\} = 0$$

Market-clearing Condition

The difference between total sales and the generation (of all producers) at node i must equal the transmission service provision:

(*12*) $$\sum_f \left(s_{if} - g_{if} \right) = y_i, \forall i \in N.$$

Consequently, if sales are greater (less) than generation at node i then the net flow occurs into (away from) node i.

Comments on LCP Solution

The LCP solution is obtained by solving (*3-6*), (*9-11*), and (*12*) simultaneously for the primal variables. Although both the producer and the ISO problems naively assume that the w_i are fixed, the w_i are variables in the LCP; the LCP solution yields the w_i that clear the market for transmission services.

IIID.1.c: Quadratic Problem (QP)

The problem in this section is based on the work of [*7*]. It can be shown that the KKT conditions that are derived for (*13-14*) are equivalent to the conditions (*3a-b*, *4a-b*, *5a-b*, *6*, *9*, *10a-b*, *11a-b*, *12*) for the LCP in section 3.4.1. This equivalence is based on an insight first reported by Hashimoto [ ].

Objective Function

The conditions that implicitly define an equilibrium with oligopolistic producers, efficient transmission allocation, and no arbitrage result from choosing the primal decision variables, s_{if} , g_{if} , and y_i, to maximize the objective [Φ^I_{QP}]:

$$(13) \qquad \Phi^I_{QP} = \sum_i \left[P_i^0 \sum_f s_{if} - \frac{P_i^0}{2Q_i^0} \left(\sum_f s_{if} \right)^2 - \frac{P_i^0}{2Q_i^0} \left(\sum_f s_{if}^2 \right) - \sum_f C_{if} g_{if} \right],$$

subject to the constraints

$$(14a) \qquad g_{if} \le G_{if}, \forall i, f. \qquad (\gamma_{if})$$

$$(14b) \qquad \sum_i s_{if} = \sum_i g_{if}, \forall f. \qquad (\alpha_f)$$

$$(14c) \qquad \sum_i PDF_{ik} y_i \le T_{k+}, \forall k. \qquad (\lambda_{k+})$$

$$(14d) \qquad -\sum_i PDF_{ik} y_i \le T_{k-}, \forall k. \qquad (\lambda_{k-})$$

$$(14e) \qquad \sum_f (s_{if} - g_{if}) = y_i, \forall i. \qquad (w_i)$$

$$(14f) \qquad s_{if} \ge 0, g_{if} \ge 0.$$

Comments on QP Solution

Prices and profits that result from the solution to this problem are unique.Generator outputs (g_{if}) may not be unique since alternative dispatches may exist that yield the same generation levels and costs for a given producer. Uniqueness results for the LCP are based on theoretical studies [Cottle]. Uniqueness results for the QP are based on [Metzler PhD].

The transmission fees (w_i) are the dual variables for the market-clearing constraints (*14e*).

Example

[To be completed.]

IIID.2: Cournot Model IIb: Exogenous Arbitrage

This model is based on the work of [7,10]. The QP formulation is derived by Michael Blake (Drayton Analytics Pty Ltd, Australia) and Benjamin Hobbs (The Johns Hopkins University, U.S.A.).

IIID.2.a: Description and Model-specific Assumptions

In *Model IIb*, power flows on a linearized DC network, and an ISO allocates scarce transmission capacity efficiently. Trade in the electricity market occurs consistent with the Bilateral Model, such that producers, *i.e.* generators, contract directly with consumers for power supply. Unlike *Model I*, however, arbitragers exist in the energy market. The arbitragers behave as price-takers, given the prices at the various nodes in the network. With the possibility of arbitrage, only transmission cost-based differences occur across nodes in the network in equilibrium.

The following assumptions on producer, consumer, and transmission behaviour characterise *Model IIb*:

- power flows over a linearized DC network;
- oligopolistic producers behave as Cournot suppliers in a Bilateral Model;
- transmission capacity is allocated efficiently; and
- arbitragers exist in the power market and behave as price-takers.

Specific to this model, producers assume naively that they are able to affect prices in one location without affecting prices at other locations in the network. In other words, producers simply accept the arbitrage quantities as given in their profit maximization problems. This modelling assumption is equivalent to a Cournot conjecture in relation to arbitrage quantities since the latter are effectively exogenous to the producers' decisions. This assumption leads to several modelling simplifications that make computation tractable for large networks.

IIID.2.b: Linear Complementarity Problem (LCP)

The collection of conditions that define the LCP solution to *Model IIb* derives from: i) the producer problem, ii) the ISO problem, iii) the arbitrager problem, and the iv) market-clearing equations.

Producer Problem

The individual producer problem is generalized from *Model I*. In *Model IIa: Endogenous Arbitrage* (not presented), producers, in their profit maximization decisions, take into account that, due to the possibility of arbitrage, their manipulation of price at one node affects prices at other nodes in the network. In this model, *Model IIb*: *Exogenous Arbitrage*, producers naively assume that they can affect prices at one node without affecting them at other nodes in the network. As a result, the arbitrage quantities do not enter the individual producer's problem as a decision variable.

In addition, consumption at each node must account for the arbitrage quantities. Specifically, with arbitrage and market-clearing, the quantity demanded at node i will equal the producer (generator) sales at node i plus the arbitrage quantities, *i.e.* $q_i = \sum_f s_{if} + a_i, \forall i$. The inverse demand function for each node must be adjusted to reflect this change from *Model I*. Given these differences, a producer, $\overline{f} \in F$, chooses generation ($g_{i\overline{f}}$) and sales ($s_{i\overline{f}}$), with the arbitrage quantities (a_i), the sales of other producers ($s_{if}, f \neq \overline{f}$), and the transmission fees (w_i) fixed, to maximize its profit $[\Pi_{\overline{f}}^{IIb}]$,

$$(15) \qquad \Pi_{\overline{f}}^{IIb} = \sum_i \left[P_i^0 - \frac{P_i^0}{Q_i^0}\left(\sum_f s_{if} + a_i\right) - w_i \right] s_{i\overline{f}} - \sum_i \left(C_{i\overline{f}} - w_i\right) g_{i\overline{f}}$$

subject to the constraints

$$(15a) \qquad g_{i\overline{f}} \leq G_{i\overline{f}}, \forall i \in N \qquad (\gamma_{i\overline{f}})$$

$$(15b) \qquad \sum_i \left(s_{i\overline{f}} - g_{i\overline{f}}\right) = 0 \qquad (\alpha_{\overline{f}})$$

$$(15c) \qquad s_{i\overline{f}} \geq 0, g_{i\overline{f}} \geq 0, \forall i \in N .$$

The KKT conditions for $[\Pi_{\overline{f}}^{IIb}]$ are:

- $s_{i\overline{f}}, \forall i$:

$$(16a) \qquad \left[\left(P_i^0 - \frac{P_i^0}{Q_i^0}\left(2s_{i\overline{f}} + \sum_{f \neq \overline{f}} s_{if} + a_i\right)\right) - w_i\right] - \alpha_{\overline{f}} \leq 0$$

$$(16b) \qquad s_{i\overline{f}} \geq 0 \qquad s_{i\overline{f}} \left\{\left[\left(P_i^0 - \frac{P_i^0}{Q_i^0}\left(2s_{i\overline{f}} + \sum_{f \neq \overline{f}} s_{if} + a_i\right)\right) - w_i\right] - \alpha_{\overline{f}}\right\} = 0$$

- $g_{i\overline{f}}, \forall i$:

$$(17a) \qquad -\left(C_{i\overline{f}} - w_i\right) - \gamma_{i\overline{f}} + \alpha_{\overline{f}} \leq 0$$

$$(17b) \qquad g_{i\overline{f}} \geq 0 \qquad g_{i\overline{f}} \left\{-\left(C_{i\overline{f}} - w_i\right) - \gamma_{i\overline{f}} + \alpha_{\overline{f}}\right\} = 0$$

- $\gamma_{i\overline{f}}, \forall i$:

$$(18a) \qquad g_{i\overline{f}} \leq G_{i\overline{f}}$$

$$(18b) \qquad \gamma_{i\bar{f}} \geq 0 \qquad\qquad \gamma_{i\bar{f}}\left(g_{i\bar{f}} - G_{i\bar{f}}\right) = 0$$

- $\alpha_{\bar{f}}$:

$$(19) \qquad \sum_i s_{i\bar{f}} = \sum_i g_{i\bar{f}}$$

ISO Problem

The ISO problem is the same as in *Model I.*

Arbitrager Problem

An arbitrager maximizes its profit by buying and selling power in the market, given prices at the nodes in the network and its costs. This model assumes that the arbitragers are price-takers and will sell power from node i to node j if $p_i(q_i) - w_i + w_j < p_j(q_j)$. The variable, $a_i > 0$, denotes the quantity sold by an arbitrager to buyers at node i, while $a_i < 0$ implies $|a_i|$ is purchased by the arbitrager at node i (and sold at other nodes). In the former case, an arbitrager receives the nodal price p_i and pays the transmission fee, w_i, per unit for the transmission of power to node i. In the latter case, an arbitrager pays the nodal price p_i and pays -w_i per unit for shipment of power from node i.

The representative arbitrager chooses a_i , given the prices, p_i , and its costs, w_i , to maximize its profit:

$$(20) \qquad \Pi_{AR}^{IIb} = \sum_i (p_i - w_i) a_i,$$

subject to the constraint

$$(21) \qquad \sum_i a_i = 0. \qquad\qquad (p_H)$$

The solution to this problem is given by the conditions:

$$(22) \qquad P_i^0 - \frac{P_i^0}{Q_i^0}\left(\sum_f s_{if} + a_i\right) - p_H - w_i = 0, \forall i \in N$$

$$(23) \qquad \sum_i a_i = 0$$

The arbitrager's KKT for a_i is $p_i - w_i - p_H = 0$. Substituting this condition into the demand for p_i yields (*22*). Constraint (*22*) ensures that price differences between nodes equal the transmission cost. Constraint (*23*) implies that arbitragers are neither net producers nor consumers.

Market-clearing Condition

The market-clearing condition must be modified to account for the possibility of arbitrage. Net total sales at node i plus arbitrage sales at node i must equal the transmission service provision to node i:

$$\sum_{f \in F} (s_{if} - g_{if}) + a_i = y_i, \forall i. \quad (24)$$

Comments on LCP Solution

The LCP solution can be obtained by solving (*16-19*), (*9-11*), and (*22-24*) simultaneously for the primal variables. Both the producers and the ISO assume that the w_i are fixed. The w_i, however, are variables in the LCP, and its solution yields the w_i that clear the market for transmission services.

IIID.2.c: Quadratic Problem (QP)

Objective Function

Alternatively, the LCP solution can also be obtained by solving a QP, the KKT conditions for which are equivalent to the LCP first-order conditions. The conditions that implicitly define an equilibrium with oligopolistic producers, efficient transmission allocation, and competitive arbitrage result from choosing the primal decision variables, s_{if} , g_{if}, y_i , and a_i, to maximize the objective [Φ_{QP}^{IIb}]:

$$\Phi_{QP}^{IIb} = \sum_i \left[P_i^0 \left[\left(\sum_f s_{if} \right) + a_i \right] - \frac{P_i^0}{2Q_i^0} \left[\left(\sum_f s_{if} \right) + a_i \right]^2 - \frac{P_i^0}{2Q_i^0} \left[\sum_f s_{if}^2 \right] - \sum_f C_{if} g_{if} \right], \quad (25)$$

subject to the constraints

$$g_{if} \le G_{if}, \forall i, f. \quad (26a) \qquad (\gamma_{if})$$

$$\sum_i s_{if} = \sum_i g_{if}, \forall f. \quad (26b) \qquad (\alpha_f)$$

$$\sum_i PDF_{ik} y_i \le T_k^+, \forall k. \quad (26c) \qquad (\lambda_{k+})$$

$$-\sum_i PDF_{ik} y_i \le T_k^-, \forall k. \quad (26d) \qquad (\lambda_{k-})$$

$$\sum_i a_i = 0 \quad (26e) \qquad (\beta)$$

$$\sum_f (s_{if} - g_{if}) = y_i - a_i, \forall i. \quad (26f) \qquad (w_i)$$

$$s_{if} \ge 0, g_{if} \ge 0, \forall i, f. \quad (26g)$$

Comments on QP Solution

This model yields the same equilibrium prices, producer outputs, and profits as a POOLCO Model with Cournot competition, in which an ISO buys power from producers and resells it to consumers through the use of a centralised auction.

Example [10]

In this example, there are three (3) nodes, two (2) firms, and three (3) unconstrained lines. The following tables summarise the remaining data for the example.

Table 1: Generator Costs and Capacities

[Node, Firm]	C_{if}	G_{if}
[1,1]	15	1000
[2,1]	15	500
[3,1]	15	0
[1,2]	20	0
[2,2]	20	1000
[3,2]	20	0

Table 2: PDF Values and Demand Parameters

Node / Line	**[1,2]**	**[1,3]**	**[2,3]**	P_i^0	Q_i^0
1	-.33	-.67	-.33	40	500
2	+.33	-.33	-.67	35	400
3	0	0	0	32	620.16

Tables 3 and 4 contain the solution values (slightly rounded) from the QP.

Table 3: Producer Generation and Sales

	Variable	***Model IIb* Results**
Generation	g_{11}	90.58
	g_{21}	271.77
	Total	362.35
	g_{22}	145.82
Sales	s_{11}	104.59
	s_{21}	95.62
	s_{31}	162.15
	Total	362.36
	s_{12}	42.09
	s_{22}	38.48
	s_{32}	65.25
	Total	145.82

Table 4: Arbitrage Quantities, Demands, and Profits

	Variable	*Model IIb* Result
Arbitrage	a_1	61.24
	a_2	-1.15
	a_3	-60.09
Prices	p_1	23.37
	p_2	23.37
	p_3	23.37
Demands	Node 1	207.91
	Node 2	132.95
	Node 3	167.31
Profits	Firm 1	3031.81
	Firm 2	490.94

V: References

1. Bai, X., S. M. Shahidehpour, V. C. Ramesh, and E. Yu (1997). "Transmission Analysis by Nash Game Method," *IEEE Trans. Power Systems*, 12(3): 1046-1052.

2. Boucher, J. and Y. Smeers (2001). "Alternative Models of Restructured Electricity Systems, Part 1: No Market Power," *Operations Research* 49(6): 821-838.

3. Cardell, J., C. Hitt, and W. Hogan (1997). "Market Power and Strategic Interaction in Electricity Networks," *Resources and Energy Economics*, 19: 109-137.

4. Chao, H. and S. C. Peck (1997). "An Institutional Design for an Electricity Contract Market and Central Dispatch," *The Energy Journal*, 18(1): 85-110.

5. Chao, H. and S. C. Peck (1996). "A Market Mechanism for Electricity Power Transmission," *Journal of Regulatory Economics*, 10(1): 25-60.

6. Cottle, R., J. Pang, and R. Stone (1992). *The Linear Complementarity Problem*, Boston, MA: Academic Press.

7. Daxhelet, O. and Y. Smeers (2001). "Variational Inequality Models of Restructured Electricity Systems," in *Complementarity: Applications, Algorithms and Extensions* (M. Ferris, O. Mangasarian, and J. Pang, *eds.*), Dordrecht: Academic Publishers, 85-120.

8. Green, R. (1996). "Increasing Competition in the British Electricity Spot Market," *Journal of Industrial Economics*, 44: 205-216.

9. Hashimoto, H. (1985). "A Spatial Nash Equilibrium Model," in *Spatial Price Equilibria: Advances in Theory, Computation, and Application*, P. Harker (*ed.*), New York: Springer-Verlag.

10. Hobbs, B. (2001). "Linear Complementarity Models of Nash-Cournot Competition in Bilateral and POOLCO Power Markets," *IEEE Transactions on Power Systems*, 16: 194-202.

11. Hobbs, B., C. Metzler, and J. Pang (2000). "Strategic Gaming Analysis for Electric Power Networks: An MPEC Approach," *IEEE Transactions on Power Systems*, 15(2): 638-645.

12. Hobbs, B. and R. Schuler (1985). "Assessment of the Deregulation of Electric Power Generation Using Network Models of Imperfect Spatial Competition," *Papers of the Regional Science Association*, 57: 75-89.

13. Hogan, W. (1992). "Contract Networks for Electric Power Transmission," *Journal of Regulatory Economics*, 4(3): 211-242.

14. Metzler, C., B. Hobbs, and J. Pang (2002). "Nash-Cournot Equilibria in Power Markets on a Linearized DC Network with Arbitrage: Formulations and Properties," *Networks and Spatial Theory* (forthcoming).

15. Metzler, C. (2000). *Complementarity Models of Competitive Oligopolistic Electric Power Generation Markets*, Ph.D. Dissertation, Dept. of Mathematical Sciences, The Johns Hopkins University, Baltimore, MD.

16. Oren, S. S. (1997). "Economic Inefficiency of Passive Transmission Rights in Congested Electricity Systems with Competitive Generation," *The Energy Journal*, 18(1): 63-83.

17. Ring, B. J. and E. G. Read (1996). "A Dispatch Based Pricing Model for the New Zealand Electricity Market," in *Electricity Transmission Pricing and Technology*, M. Einhorn and R. Siddiqi, *eds.*, Boston: Kluwer Academic Publishers.

18. Schweppe, F., M. Caramanis, R. Tabors, and R. Bohn (1988). *Spot Pricing of Electricity*, Boston: Kluwer Academic Publishers, Appendix D.

19. Smeers, Y. and J. Wei (1997). “Do We Need a Power Exchange If There Are Enough Power Marketers?,” *CORE Discussion Paper 9760*, Universite Catholique de Louvain.

20. Smeers, Y. and J. Wei (1997). “Spatially Oligopolistic Model with Opportunity Cost Pricing for Transmission Capacity Reservations – A Variational Inequality Approach,” *CORE Discussion Paper 9717*, Universite Catholique de Louvain.

21. von der Fehr, N. and D. Harbord (1993). “Spot Market Competition in the UK Electricity Industry,” *Economic Journal*, 103: 531-546.

22. Wu, F. F. (1996). “Coordinated Multilateral Trades for Electric Power Networks,” Department of Electrical Engineering, University of Hong Kong.