Game Theory and Electricity Markets

Basic Game Theory Concepts and Applications to Electricity Markets

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I: Introduction

Many countries worldwide have initiated the privatisation of electricity assets, the restructuring of electricity industries, and the implementation of the technical, legal, and institutional structures to support the development of electricity markets. The objective of these activities has been to create a competitive environment for trade in electricity that ultimately produces net benefits to society.

The realisation of these potential benefits, however, depends on the unfettered development and efficient operation of markets, which includes (among other factors) limiting the opportunities for participants to exercise market power. Although the definition and measurement of market power are subject to persistent debate, a participant, in general, possesses market power when it is able to determine unilaterally key components of its market conduct without any constraints. The actual exercise of market power typically involves a firm withholding some of its output and charging prices above marginal cost for a sustained period of time. Although this characterisation is general and not without qualification, a more detailed discussion of market power is beyond the scope of this article at present.

The exercise of market power creates productive and allocative inefficiencies, including i) too little production relative to the competitive level, ii) too little consumption relative to the competitive level, and iii) higher prices relative to competitive levels. Market power may also create dynamic inefficiencies if it distorts price signals for new investment. As a result, the effects of market power may erode the benefits that would otherwise arise from a fully competitive market. In addition, the exercise of market power leads to a transfer of inframarginal revenues from consumers to producers (but this transfer is not an inefficiency itself).

In electricity markets, the physical properties of electricity, economic aspects of electricity supply and demand, and market design/institutional features may create opportunities for the exercise of market power by participants. Such aspects include, but are not limited to, the:

- market structure - number of generation companies in the market and the degree of vertical integration of these firms;
- supply 'tightness' relative to demand in certain periods, *i.e.* the amount of residual demand facing firms in the market in a given period;
- elements of market design, including the
  - design of the market-clearing mechanism (centralised auction versus bilateral trade),
  - transmission pricing methodology, and
  - operational rules and procedures;
- topology of the transmission network, including the location of generation and loads, and the potential for transmission congestion; and the
- prevailing regulatory structure and. policies.

Recent empirical studies provide some evidence that generators have exercised market power in both the California and United Kingdom (U.K.) power markets [5,20]. These studies, in combination with the physical, economic, and design aspects of electricity markets, have continued to motivate market power research. The economics and operations research fields feature a wide range of approaches to analysing the exercise of market power in electricity markets. These approaches include equilibrium models of imperfect competition, simulation studies, and laboratory experiments with artificial agents.

The most prevalent approach to date is the equilibrium model of imperfect competition. This approach is promising for several reasons. Analysis of market power seeks to answer basic questions, ranging from how a firm exercises market power and under what conditions to how market power impacts equilibrium quantities, prices, and social welfare in an economy. First, attempting to answer these questions requires accurately characterising the relevant market and its institutional features. Market characteristics include, for example, the number of firms in the relevant industry and their cost structures, the degree of product differentiation, and the nature of interaction among these firms. Institutional features include, for example, the legal and regulatory frameworks that determine the organisation and operation of the market. Since knowledge of these specific factors is essential to establishing a context for answering the questions posed, it is obviously critical to represent these features in the analytical approach. Computable equilibrium models, in general, offer sufficient flexibility for representing these market and institutional features.

Second, answering these questions always involves assessing the potential impact of market power on economic variables, such as prices and quantities. This assessment, at least implicitly, requires a comparison of *ex post* observations of real world market prices and quantities to the comparable values of these variables that the perfectly competitive model predicts. Although perfect competition may characterise few real world markets, its importance primarily relates to serving as a benchmark. The story does not end at this point, however, since the observed data may depart significantly from the competitive benchmark values due to market and institutional features that are *independent* from the exercise of market power. Computable equilibrium models enable the analyst to build a model that includes these features to the required level of detail, such that deviations from the benchmark are captured by parameters in the model. At a basic level, this approach is consistent with the view of an equilibrium model of imperfect competition as a model of perfect competition distorted by quantity restrictions and price mark-ups above marginal cost [13].

In applying these models, it is helpful to understand the relationship between imperfectly competitive markets and other market types. In general, it is possible to categorise markets into different classes, depending on the intensity of competition that, in principle, prevails within them. Figure 1 illustrates the relationship between the level of competition and market type.

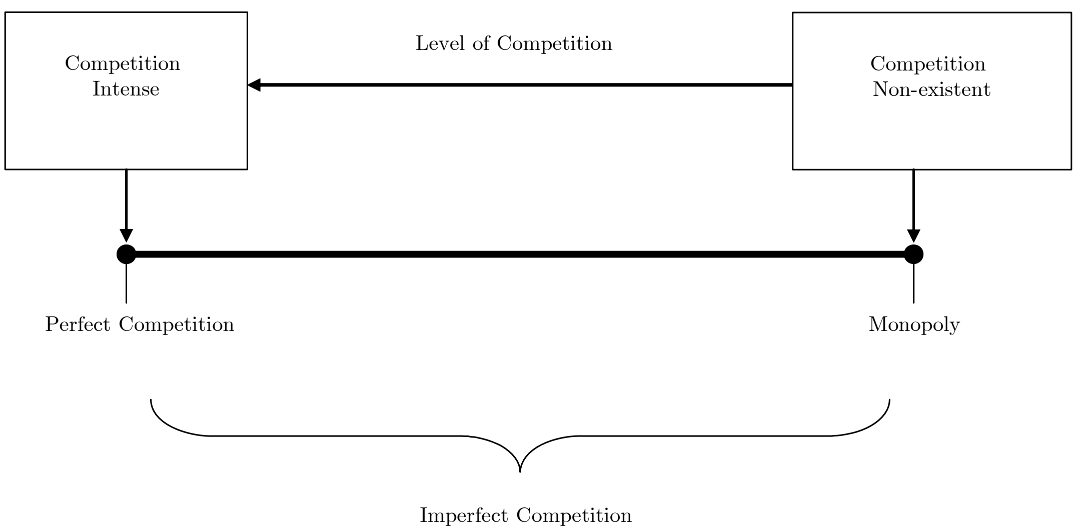


Figure 1: The Degree of Competition in Markets

At one extreme is perfect competition, characterised by i) product homogeneity, ii) full resource mobility, iii) perfect information, and iv) price-taking behaviour by participants. Price-taking behaviour implies that each individual producer (buyer) in a market, when choosing its production (purchases), assumes that its choice will have no impact on the aggregate demand-supply balance and consequently, no impact on the market price. (Note that a competitive producer, although a 'price-taker', is at the same time a 'price-adjuster', in the sense that it responds to excess demand or supply by increasing or decreasing (respectively) its price.) A perfectly competitive firm's marginal revenue, therefore, is the market price, and profit-maximising behaviour results in producing the output level at which price equals marginal cost. In this case, the actions of other firms are largely irrelevant to an individual firm's profit maximisation decision. Markets for agricultural commodities probably are the best approximation to perfect competition.

At the opposite extreme, a monopoly consists of a single producer that faces the entire market demand for a product. Entry into this market type is difficult due to barriers that impede the entry of new competitors, such as economies of scale, technology patents, or the monopolist's ability to control access to essential inputs to production. The monopolist maximises its profit by producing the output level at which its marginal revenue equals marginal cost and charging a price above the socially optimal price, which is marginal cost. The mark-up, *i.e.* the ratio between its profit margin (price less marginal cost) and the price, is inversely proportional to the elasticity of market demand.

Imperfectly competitive markets lie between these two extremes and are divisible into two basic market structures, monopolistic competition and oligopoly. Monopolistically competitive markets are characterised by many producers, differentiated products, and free entry and exit. Oligopoly markets are characterised by a limited number of producers, homogeneous or differentiated products, barriers to entry, and strategic interdependence among firms. The focus in this article is on the oligopoly market structure since electricity production tends to possess scale economies, which leads to barriers to entry and a limited number of competing firms.

A central feature of oligopoly is that a few large firms in the market dominate production and are able to exercise market power by altering their output and/or pricing decisions to their advantage. Due to barriers to entry, some or all of these firms may earn substantial, positive economic profits over a sustained period of time. Since the number of firms is limited, each individual firm must consider its own set of market actions, *e.g.* output and pricing decisions, and the impact of these actions on its rivals. Further, each individual firm must account for possible reactions of rivals to its actions and the fact that its rivals will make a similar assessment of their own.

A prominent issue in the industrial organisation literature, therefore, is how an individual firm accounts for, and responds to, its rivals' actions in an imperfectly competitive market. In examining this issue, a complementary question that naturally arises is given a theory about an individual firm's behaviour toward its rivals and their possible reactions, how should such behaviour be modelled from an analytical perspective. One plausible answer to this question begins with the assumption that a 'rational' firm acts to maximise its profit. This assumption is known as the profit maximisation hypothesis and serves as a central postulate for economic theory. (Some theories of firm behaviour offer alternative hypotheses, but this subject is beyond the current scope of this article – see Tirole [19] for a basic overview of the relevant literature.)

It follows from the profit maximisation hypothesis that an individual firm's profit maximisation decision should encapsulate some assumption regarding how its rivals will react to its output/pricing decisions. As a result, the primary component of models of imperfect competition is a specification of how a firm assumes that its rivals (possibly including potential entrants to the market) react to its decisions. The dominant approach in the economics literature for examining this strategic interaction among firms in an imperfectly competitive industry is game theory.

II: Basic Game Theory Concepts

This section introduces basic concepts in game theory that serve as the building blocks for equilibrium models of imperfect competition. For a comprehensive discussion of game theory concepts applied to economics, see Gibbons [11] and for a more advanced treatment, see Fudenberg and Tirole [10].

The examples in this section are adapted from [11].

II.A: The Strategic Form

Game theory is the study of multi-person or multi-firm decision-making problems. In the field of industrial organisation in economics, game theory is used extensively to study auction behaviour, bargaining, principal-agent relationships, product differentiation, and strategic behaviour by firms.

The strategic (or normal) form representation of a game includes three components:

- the set of players, $i \in \{1,...,I\}$, in the game, which is assumed finite;
- the pure strategy space, S_i, which contains the individual strategies available to player i, $s_i \in S_i$, where s_i is an arbitrary strategy; and
- the pay-off function, $u_i(s)$, which gives player i's von Neumann-Morgenstern utility for each profile $s = (s_1,\ldots,s_I)$ of strategies that could be played in the game, with $\overline{S}$ representing the space of profiles ($s \in \overline{S}$).

In a game, a player often takes the form of a rational individual or a profit-maximising firm. Each player's objective is not to 'defeat' the other players (denoted $-i$), *i.e.* its rivals, but to maximise its pay-off function from playing the game. Playing the game to achieve this objective may 'benefit' or 'harm' other players in the game.

In a static game, a pure strategy is simply an action, and a player's pure strategy space is a set of possible actions. For example, Player 1 in a game may possess the (pure) strategy space, S_1 = {Attack, Retreat}, and these are the only possible actions from which the player can choose a strategy in the game. In a dynamic (repeated) game, a strategy is a complete plan of action that specifies a possible action for the player for each contingency in the game for which the player may be compelled to play. In economics, the most common strategy variables are quantity and price, while in political science, the strategy variable may represent votes. Further, a strategy may either be a pure or mixed strategy. For purpose of exposition, the remainder of this article focuses on the play of pure strategies, although mixed strategies are possible equilibria in many games (see the discussion in section II.B for a basic overview of mixed strategies).

The pay-off for a player may be a measure of personal utility or income, and the pay-off for a firm may be profit, reputation, *etc.*

The following example illustrates the use of the notation defined previously. Assume that the strategic form of a game consists of two players (i =1,2), each with two possible strategies. Player 1 can choose to play 'Up' or 'Down', while Player 2 can choose to play 'Left' or 'Right'. The strategy spaces for the players, therefore, are: S_1 = {Up, Down}, S_2 = {Left, Right}. One possible profile of strategies is that Player 1 plays 'Up', and Player 2 plays 'Left', *i.e.* s = (Up, Left). If Player 1 receives a pay-off of 1 and Player 2 receives a pay-off of 0 from the play of this profile of strategies then $(u_1(s), u_2(s)) = (1,0)$. Finally, the space of profiles is $\overline{S}$ = {(Up, Left); (Up, Right); (Down, Left); (Down, Right)}.

Given the strategic form of a game, the next section introduces the simplest type of game and proposes two different solution concepts for obtaining an outcome from the play of such games.

II.B: Basic Solution Concepts

A basic and useful game is a *static game of complete information*. These games have a simple but powerful form:

- the structure of the game is *common knowledge* - all players know the strategic form of the game, know that their opponents know it, know that their opponents know that they know it, *etc.*, *ad infinitum* - consequently, each player has common knowledge of the other players, their strategies, and their pay-off function (for a formal definition of common knowledge see [2]);
- the players *simultaneously* choose their strategies – players do not necessarily act simultaneously, but each player chooses its strategy without knowing the strategy *choice* of the other players; and
- after playing the game, each player receives a pay-off that depends on the profile of strategies played.

The following example of a static game of complete information builds on the example given previously. Recall that each player has two strategies in its strategy space: S_1 = {Up, Down}, S_2 = {Left, Right}. These strategies and the associated pay-offs are illustrated with the 'bi-matrix' in Figure 2. Each pay-off cell in the matrix corresponds to the play of a particular profile of the players' strategies and contains a pair of numbers that represents the pay-offs to the players. The first (second) listed number gives the pay-off to Player 1 (2) that is associated with that particular profile of strategies. For example, if Player 1 plays 'Up' and Player 2 plays 'Left', then the pay-offs are 1 to Player 1 and 0 to Player 2. (The pay-offs typically are either measured in dollars or utility, but they could represent any desired metric.)

		Player 2	
		Left	Right

Player 1	Up	1,0	1,2
	Down	0,3	0,1

Figure **2***: Example – Iterated Elimination of Strictly Dominated Strategies*

The relevant issue is how to obtain a solution to this game-theoretic problem, *i.e.* what profile of players' strategies represents a 'likely' outcome of this game. A useful starting point is the idea that rational players will not play 'strictly dominated' strategies. In this game, Player 1 will never play 'Down': if Player 2 plays 'Left' then Player 1 plays 'Up' since $1 > 0$, and if Player 2 plays 'Right' then Player 1 again plays 'Up' since $1 > 0$. Regardless of the choice by Player 2, Player 1 always maximises its pay-off by playing 'Up'; 'Down', therefore, is a strictly dominated strategy. Given that Player 2 correctly anticipates that a rational Player 1 will never play 'Down', Player 2 knows that Player 1 will always choose 'Up'. Player 2 correctly anticipates this choice and chooses 'Right' since $2 > 0$. As a result, the solution of the game is (Up, Right), obtained by the *iterated elimination of strictly dominated strategies*. For games in which players have more strategies, there may be successive 'rounds' of eliminating strictly dominated strategies (hence, the 'iterated' in the solution name).

Although the concept that rational players do not play strictly dominated strategies is sound, it suffers from two major weaknesses. First, each step of the game assumes that all players are rational, which in aggregate, *i.e.* across all steps, results in the assumption of common knowledge. The second weakness is that the application of this solution method does not produce a solution for all games. If all strategies in a game survive the iterated elimination of strictly dominated strategies then the method predicts nothing about the play of the game.

An equilibrium concept that produces much stronger predictions about the play across a wide range of games is *Nash equilibrium* [17]. As a thought experiment, an excellent motivation for Nash equilibrium as a solution concept is that if game theory provides a unique prediction about the strategy that each player will choose, then in order for this prediction to be correct, it is necessary that each player is willing to choose the strategy that the theory predicts. In other terms, if the play of a game is defined by a pact among all of the players then the strategies that the pact prescribes to the players must define a Nash equilibrium; otherwise, at least one player will want to deviate from the pact, and it will not be binding.

More specifically, a strategy is a Nash equilibrium for a player if that player cannot increase its own pay-off by undertaking any strategy other than its equilibrium strategy, given the strategy choices of its rivals. In a Nash equilibrium, each player will decrease its pay-off if it deviates from its Nash equilibrium strategy, assuming all other players continue to play their existing strategies. As a result, a Nash equilibrium is a 'best response', in the sense that no player has an incentive to deviate from its strategy choice, given all other players' strategy choices. Definition 1 gives a formal definition of Nash equilibrium.

Definition 1: In the n-player strategic form game, G = $\{S_1,\ldots,S_n;\ u_1,\ldots,u_n\}$, the strategies $(s_1^*,\ldots,s_n^*)$ are a Nash equilibrium if, for each player i, s_i^* is player i's best response to the strategies specified for the other (n-1) players, $(s_1^*,\ldots,s_{i-1}^*,s_{i+1}^*,\ldots s_n^*)$, such that $u_i(s_1^*,\ldots,s_{i-1}^*,s_i^*,s_{i+1}^*,\ldots,s_n^*) \geq u_i(s_1^*,\ldots,s_{i-1}^*,s_i,s_{i+1}^*,\ldots,s_n^*)$, for every feasible strategy $s_i \in S_i$.

Nash's Theorem is proved through the use of a fixed-point theorem. As an example, suppose that $f(x)$ is a continuous function with domain [0,1] and range [0,1]. Brouwer's Fixed-point Theorem ensures that at least one fixed-point exists, *i.e.* there exists at least one value x^* in [0,1] such that $x^* = f(x^*)$. The use of a fixed-point theorem to prove Nash's Theorem requires two steps: i) showing that any fixed-point of a certain correspondence is a Nash equilibrium, and ii) applying Kakutani's Fixed-point Theorem to show that this correspondence must have a fixed-point. Kakutani's Fixed-point Theorem is the relevant fixed-point theorem because it generalises Brouwer's Fixed-point Theorem to allow for well-behaved correspondences as well as functions. Although demonstrating these two steps is beyond the scope of this article, [10] contains the relevant proofs.

Figure 3 illustrates the concept of Nash equilibrium through a simple game.

		Player 2		
		Left	Middle	Right
Player 1	Up	1,0	1,2	0,1
	Down	0,3	0,1	2,0

Figure **3***: Example - Game with a Unique Nash Equilibrium*

The solution to this game is determined by finding the profile(s) of strategies such that each player's strategy is a best response to the other player's predicted strategy. For example, consider the profile, (Up, Left). Given Player 1 plays 'Up', Player 2's best response is to play 'Middle' since $2 > 0$. Consequently, 'Left' is not a best response for Player 2, and (Up, Left) cannot be a Nash equilibrium since Player 2 would deviate from it. Now consider the profile, (Down, Left). Given Player 1 plays 'Down', Player 2's best response is to play 'Left' since $3 > 1$ and $3 > 0$. Consequently, Player 2 would not want to deviate. Player 1, however, would deviate from it: given Player 2 plays 'Left', Player 1's best response is to play 'Up' since $1 > 0$. (Down, Left), therefore, is not a Nash equilibrium either. The same analysis can be applied to the other cells, in turn, and the unique Nash equilibrium is (Up, Middle): each player's strategy choice is a best response.

The following two propositions define the relationship between Nash equilibrium and the iterated elimination of strictly dominated strategies:

Proposition 1: In the n-player strategic form game, $G = \{S_1,\ldots,S_n;\ u_1,\ldots,u_n\}$, if iterated elimination of strictly dominated strategies eliminates all but the strategies, $(s_1^*,\ldots,s_n^*)$, then these strategies are the unique Nash equilibrium of the game.

By Proposition 1, the equilibrium, (Up, Right), in Figure 2, obtained through the iterated elimination of strictly dominated strategies, is a Nash equilibrium.

Proposition 2: In the n-player strategic form game, $G = \{S_1,\ldots,S_n;\ u_1,\ldots,u_n\}$, if the strategies, $(s_1^*,\ldots,s_n^*)$, are a Nash equilibrium then they survive iterated elimination of strictly dominated strategies.

In Figure 3, it was shown previously that (Up, Middle) is the unique Nash equilibrium. Consider a solution to this game using the iterated elimination of strictly dominated strategies. For Player 2, the strategy, 'Right', is strictly dominated by 'Middle', *i.e.* if Player 1 plays 'Up' then $2 > 1$, and if Player 1 plays 'Down' then $1 > 0$, so Player 2 never plays 'Right'. This round of iterated elimination, therefore, eliminates the strategy, 'Right', for Player 2, reducing the game in Figure 3 to the exact game in Figure 2. As shown previously, the game in Figure 2 is solvable by the iterated elimination of strictly dominated strategies, and the equilibrium is (Up, Middle). Consequently, this Nash equilibrium survives the iterated elimination of strictly dominated strategies, consistent with Proposition 2.

II.C. Mixed Strategies

A player's pure strategy is actually a special case of a mixed strategy. More specifically, assume that player i has K pure strategies, $S_i = \{s_{i1},...,s_{iK}\}$. A *mixed strategy* for player i is a probability distribution, $\sigma_i = (\sigma_{i1},...,\sigma_{iK})$, where $_{ik}(s_{ik})$ is the probability that player i will play strategy s_{ik}, for $k = 1,\ldots,K$, such that $0 \leq \sigma_{ik} \leq 1$ for $k = 1,\ldots,K$ and $\sigma_{i1} + ... + \sigma_{iK} = 1$. In a mixed strategy, each player's randomisation is statistically independent from the randomisations of its rivals, and the pay-offs to players associated with a specific profile of mixed strategies are the expected values of the corresponding pure strategy pay-offs.

Since $_i$ represents an arbitrary mixed strategy for player i, let represent a profile of mixed strategies of the players, and let represent the space of mixed strategy profiles for the game. These assumptions and notation allow the formalisation of Definition 2.

Definition 2: In the strategic form game, G = $\{S_1,\ldots,S_n;\ u_1,\ldots,u_n\}$, suppose $S_i = \{s_{i1},...,s_{iK}\}$. A mixed strategy for player i is a probability distribution, $\sigma_i = (\sigma_{i1},...,\sigma_{iK})$, such that $0 \leq \sigma_{ik} \leq 1$ for $k = 1,\ldots,K$ and $\sigma_{i1} + ... + \sigma_{iK} = 1$.

An important implication of Definition 2 is that the set of mixed strategies contains the pure strategies as degenerate probability distributions. The following example illustrates the notation in Definition 2 and this implication.

In the game in Figure 3, the players' pure strategy spaces are S_1 = {Up, Down} and S_2 = {Left, Middle, Right}. Assume now that Player 1 has the following mixed strategies: $\sigma_{11} = (.5,\ .5)$, $\sigma_{12} = (1,\ 0)$, $\sigma_{13} = (0,\ 1)$. Player 2 has the mixed strategies: $\sigma_{21} = (.3,\ .4,\ .3)$, $\sigma_{22} = (1,\ 0,\ 0)$, $\sigma_{23} = (0,\ 1,\ 0)$, $\sigma_{24} = (0,\ 0,\ 1)$. Suppose a mixed strategy profile for the players is $\sigma \in \Sigma$ = [(.5, .5); (.3, .4, .3)]. Player 1's expected pay-off from this profile is: $u_1(\ ) = .5[(.3)(1) + (.4)(1) + (.3)(0)] + .5[(.3)(0) + (.4)(0) + (.3)(2)] = .65$.

II.D. Multiple Nash Equilibria

A game may possess multiple Nash equilibria. Figure 4 illustrates this possibility.

		Player 2	
		Left	Right
Player 1	Up	2,1	0,0
	Down	0,0	1,2

Figure **4***: Example - Game with Multiple Nash Equilibria*

In this game, both (Up, Left) and (Down, Right) are Nash equilibria. It is not clear, however, which of these equilibria would prevail as a solution to the game. As a result, game theory research has devoted significant attention to developing methods for identifying a 'compelling' equilibrium in games with multiple Nash equilibria. One approach for 'filtering' multiple equilibria is the theory of 'focal points' [18]. This theory proposes that in some real world situations, players may be able to coordinate on a single, or 'focal', equilibrium through information that is independent from the strategic form. For example, suppose two players are asked to simultaneously name an exact time of day (to the minute), with the understanding that if the times match, they both receive a large, positive pay-off. A choice of 12 midnight or 12 noon is focal, but a choice of 4:17 p.m. is probably not focal. The problem with the 'focal point' approach, however, is that the degree of 'focalness' of strategies may depend on the players' past experiences and cultures. For example, a given player's strategy choice of 'Left' or 'Right' in a particular game may depend on the direction of flow of traffic in the individual's home country [10].

II.E: Applications to Microeconomics

The analysis in this section is based on Tirole [19].

II.E.1: Notation

Throughout the remainder of this article, the following notation is relevant:

q_i	output of firm i	
p_i	price of firm i	
q_{-i}	output of firm i's rival	
p_{-i}	price of firm i's rival	
Π^i	profit function of firm i	
$R_i(s_j)$	reaction function of firm i, a function of firm j's strategy	
Π_i^i	first derivative wrt s_i of firm i's profit function	$\partial \Pi^i(s_i, s_j) / \partial s_i$
Π_{ii}^i	second derivative wrt s_i of firm i's profit function	$\partial^2 \Pi^i(s_i, s_j) / \partial s_i \partial s_i$
Π_{ij}^i	cross-partial derivative of firm i's profit function	$\partial^2 \Pi^i(s_i, s_j) / \partial s_i \partial s_j$

In general, subscripts on the profit function denote partial derivatives with respect to s_i and s_j, while superscripts denote the firm. Subscripts on strategies (*e.g.* quantities and prices) and reaction functions, however, always reference the firm.

II.E.2: Reaction Functions

Each firm in an oligopoly independently and simultaneously acts to maximise its profit in a static game of complete information. The firms (i, j) have profit function $\Pi^i = (s_i, s_j)$ and $\Pi^j = (s_i, s_j)$ respectively. The first-order condition for a Nash equilibrium for firm i is:

(*1*) $\Pi_i^i(s_i^*, s_j^*) = 0$,

where the * superscript denotes the optimal strategy choice, *i.e.* the choice that satisfies the Nash equilibrium condition. The second-order condition requires that $s_i = s^*_i$ gives a local maximum for the problem, *i.e.*

(*2*) $\Pi_{ii}^i(s_i^*, s_j^*) \leq 0$.

If each firm's profit function is strictly concave in its own strategy everywhere, *i.e.*

(*3*) $\Pi_{ii}^i(s_i, s_j) < 0, \forall (s_i, s_j)$,

then the second-order condition is satisfied, and (*1*) is sufficient for a Nash equilibrium. Such an equilibrium is then defined by a system of two equations with two unknowns.

Define $R_i(s_j)$ as the 'best response' of firm i to firm j's choice of s_j:

(*4*) $\Pi_i^i(R_i(s_j), s_j) = 0$.

$R_i(s_j) = s_i$ is unique from the assumption of strict concavity and represents firm *i*'s reaction to s_j. Further, a Nash equilibrium is a set of actions (s_i^*, s_j^*), such that:

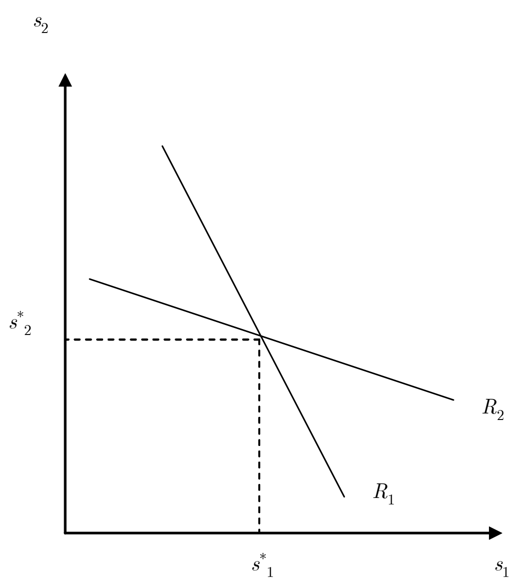
(*5a*) $s_i^* = R_i(s_j^*)$ and

(*5b*) $s_j^* = R_j(s_i^*)$.

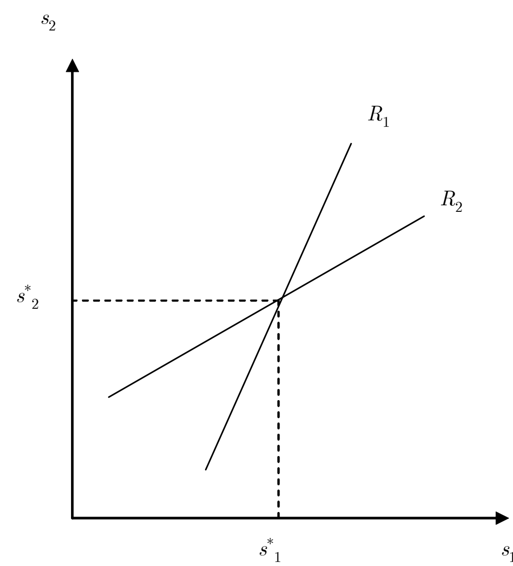
In an equilibrium defined by (*5a-b*), each firm's strategy choice is a best response to its rival's strategy choice. An important feature is the slope of a firm's reaction function. Differentiating (*4*) to obtain the slope of firm *i*'s reaction function yields:

(*6*) $$R_i^i(s_j) = \frac{\Pi_{ij}^i(R_i(s_j), s_j)}{-\Pi_{ii}^i(R_i(s_j), s_j)} .$$

Condition (*6*) implies that $sign\left(R_i^i\right) = sign\left(\Pi_{ij}^i\right)$, where the reaction function has a positive slope if $\ {}^{i}_{ij} > 0$ and a negative slope if $\ {}^{i}_{ij} < 0$. In the former case, the goods are 'strategic complements' and in the latter case, they are 'strategic substitutes' (see [6] for a complete discussion).



(i) Strategic substitutes $\Pi^i_{ij} < 0$



(ii) Strategic complements $\Pi^i_{ij} > 0$

Figure 5: Strategic Substitutes and Complements

In economics, typically prices are strategic complements, and quantities are strategic substitutes. Note that Figure 5 simply illustrates the best response of each firm to a change in its rival's strategy; however, points other than the Nash equilibrium are never actually observed since each firm chooses its action simultaneously, without observing the choice of its rival.

III: Selected Models of Imperfect Competition

The three primary equilibrium models applied to examine market power are the Cournot, Bertrand, and Supply Function Equilibrium (SFE) models of imperfect competition. This section gives a basic description of these models and discusses their application in an electricity market context.

The Bertrand and Cournot examples are adapted from [11].

III.A: Overview

The common assumption of these models is that each individual, competing firm seeks to maximise its profit given:

- demand conditions;
- its cost structure;
- any other relevant market conditions; and
- an assumption about how rivals will respond to its decisions.

The key difference among the models is the strategic variable that a firm chooses when competing against its rivals. The choice of strategy, *e.g.* price, quantity, or supply function, impacts the intensity of competition among the firms and the resulting equilibrium outcomes that the models predict. Figure 6 illustrates where the intensity of competition predicted by the basic formulation of each of the models places them along the competitive spectrum.

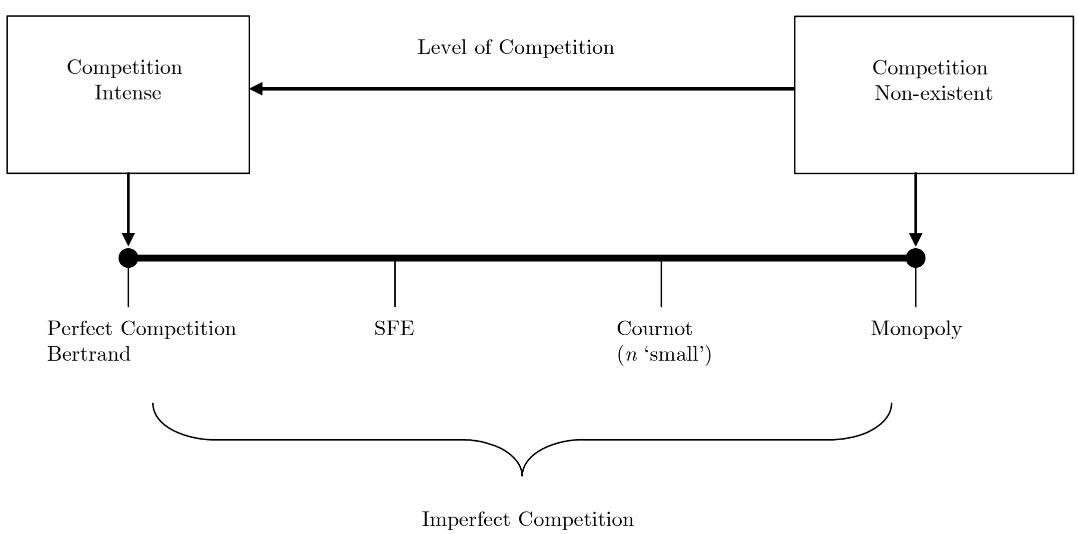


Figure 6: Equilibrium Models and Predicted Degree of Competition

In the Bertrand model, for example, players believe that their rivals will not change their prices. In the traditional Bertrand model, with homogeneous products and identical unit production costs, the Nash equilibrium outcome is the competitive outcome in which the firms charge a price equal to marginal cost in equilibrium. In the Cournot model, each individual player assumes that its output affects price but that this decision does not affect the output of its rivals. As a result, the market price is higher than the purely competitive price but less than the monopoly price (assuming that the number of firms is not too large – see Appendix A). In the SFE model, firms bid entire supply functions, and the resulting price equilibria generally are between the Bertrand and Cournot outcomes.

III.B: Bertrand

The material in section III.B.1 is adapted from [19].

III.B.1: Description

In the classic model of Bertrand duopoly, firms compete against each other using prices as strategy choices [3]. Three basic assumptions set the stage for the classic Bertrand duopoly model. First, competing firms produce homogeneous, *i.e.* identical, products, which are perfect substitutes in consumers' utility functions. Consumers purchase from the firm with the lowest price. If the firms charge the same price, the model assumes that each firm faces a demand schedule that is equal to half of the market demand. Given two firms (i, j) and the market demand $q = D(p)$, this assumption implies that the demand, $D(p_i)$, for the output of firm i is:

$$(7) \qquad D_i(p_i, p_j) = \begin{cases} D(p_i), & p_i < p_j \\ \frac{1}{2} D(p_i), & p_i = p_j \\ 0, & p_i > p_j \end{cases} .$$

Second, firms have identical unit production costs, $C(q_i) = cq_i$, and always possess the capacity to supply sufficient output to satisfy demand, *i.e.* there are no capacity constraints on production. Third, firms 'meet only once' in the market and choose prices non-cooperatively and simultaneously. The non-cooperative nature of the game implies that competing firms act in their own self-interest, and the simultaneous aspect implies that each firm chooses its price without observing the choices of its rivals. In the Bertrand model of duopoly, each firm correctly anticipates its rival's price choice, such that the price it chooses in equilibrium maximises its profit (given the price choice by its rival).

The aggregate profit of the firms cannot exceed the monopoly profit (^{m}), and each firm can ensure itself a *non-negative* profit by charging a price above marginal cost; therefore, any reasonable outcome of the model must satisfy 0 ^{1} + ^{2} ^{m}. Under the basic assumptions, the unique Nash equilibrium of the classic Bertrand model is a pair of prices, (p_i^*, p_j^*), such that each firm charges the competitive price, *i.e.* $p_i^* = p_j^* = c$.

To understand this result, consider the following cases:

i) $p_i^* > p_j^* > c$

In this case, firm *i*'s demand is zero, and its profit is zero as a result. If firm *i* instead charges $p_i = p_j^* - \varepsilon$, where ε is positive and 'small', it acquires the entire market demand and makes a positive profit margin, $p_j^* - \varepsilon - c$. Consequently, in this case, p_i^* is not a best response by firm *i* to firm *j*'s price.

ii) $p_i^* = p_j^* > c$

In this case, firm *i*'s profit is $D(p_i^*)(p_i^* - c)/2$. If firm *i*, however, reduces its price slightly to $p_i^* - \varepsilon$, its profit is $D(p_i^* - \varepsilon)(p_i^* - \varepsilon - c)$, which is higher for 'small' . Again, p_i^* is not a best response by firm *i* to firm *j*'s price in this case. Since neither firm will charge less than its unit cost (resulting in a negative profit for the lowest price firm), only two cases remain: one firm charges exactly *c*, or both firms charge exactly *c*. The former case is now shown to be false by contradiction.

iii) $p_i^* > p_j^* = c$

In this case, firm *j* supplies all of the demand and makes zero profit. If firm *j*, however, charges $p_j = p_j^* + \varepsilon$, it still retains the entire market demand and makes a positive profit, resulting in a contradiction. The Nash equilibrium in the Bertrand model, therefore, must involve both firms charging the competitive price, *i.e.* $p_i^* = p_j^* = c$.

The implications of the classic Bertrand model are that firms i) price at marginal cost and that firms ii) do not make positive, economic profits. These results constitute the Bertrand paradox because they imply that a competitive outcome occurs even in an industry with only two competitors. Interestingly, these results also suggest that the monopoly market equilibrium represents a special case (since the competitive outcome requires only two firms). A corollary to the paradox relates to entry into the market. Suppose a 'small' fixed cost ($F =$) is required for entry into the market. If one firm enters the market, the second firm will not enter as long as $F > 0$. Consequently, if one of the firms believes that $F > 0$, the outcome yields a market that is a monopoly.

The relaxation of any of the classic assumptions weakens, if not eliminates, the paradox. First, if products are identical then consumers purchase from the firm with the lowest price. Consequently, product homogeneity places downward pressure on price. If firms' products are not identical, however, then product differentiation alleviates some of this downward pressure. In this case, firms do not, in general, charge a price equal to their marginal cost of production.

Second, in the case of asymmetric unit costs, *e.g.* $c_i > c_j$, the Bertrand conclusions, in general, are not robust. Further, the existence of a strict capacity constraint represents a decreasing returns to scale technology. An example of a strict capacity constraint is a firm with a unit cost of c up to the constraint and then a cost of   for any greater output. More generally, marginal cost may increase 'significantly' with output. With capacity constraints, a firm cannot, by definition, satisfy the entire demand alone; a price equal to marginal cost is no longer an equilibrium. Suppose firms i and j charge marginal cost as in the classic solution. Suppose now that firm i faces a capacity constraint. If firm j now charges $p_j > c$ then firm i confronts all of the demand, which it is unable to satisfy. In this case, the residual consumers must purchase the product from firm j at the higher price. Firm j has a non-zero, residual demand at a price that exceeds its marginal cost, and as a result, it makes a positive profit. The classic solution, therefore, is not an equilibrium if (at least) one firm confronts a capacity constraint.

Third, in a dynamic game, which allows for reactions over time, firm j would consider the trade-off between the short run benefits of any price cuts and the long-run costs of a potential price war with firm i.

III.B.2: Example

The classic Bertrand duopoly model postulates that two duopolists in a market interact with each other through price competition. Since the previous section demonstrates that for the case of two duopolists with identical products, the unique Nash equilibrium is $p_i = p_j = c$, this example assumes differentiated products. Specifically, this game assumes that:

- two firms ($i = 1,2$) produce heterogeneous products (q_i) that are imperfect substitutes;
- demand for firm i's product is $q_i(p_i, p_{-i}) = a - p_i + bp_{-i}$, where $0 < b < 2$ is a parameter that determines the extent to which firm i's product is a substitute for its rival's product;
- total cost of production is $C_i(q_i) = cq_i$, where marginal cost is constant at c and $c < a$; and that
- firms simultaneously choose their respective prices from the feasible set, $p_i = [0, \quad)$.

These assumptions give all of the required components of the strategic form. First, the players are, of course, the two firms that comprise the duopoly. Second, the strategies available to each firm are the feasible prices, where it is assumed that negative prices are infeasible and non-negative prices are infinitely divisible. Third, each firm's strategy space is denoted $S_i = [0, \quad)$, implying that a typical strategy is a price choice, $p_i \quad 0$. Finally, it is necessary to specify the pay-off function of firm i as a function of its strategy and the strategy selected by the rival firm. A firm's pay-off is described by its profit function. The generalised pay-off to firm i, $u_i(s_1, s_2)$, applied to this two-player strategic form game, therefore, is specified as:

$$(8) \qquad \Pi^i(p_i, p_{-i}) = q_i(p_i, p_{-i})[p_i - c] = (a - p_i + bp_{-i})[p_i - c]\,.$$

Recall that the strategy pair, (s_1^*, s_2^*), is a Nash equilibrium if, for each player i,

(9) $\quad u_i(s_i^*,\ s_{-i}^*) > u_i(s_i,\ s_{-i}^*)\ ,\ \forall s_i \in S_i$.

Equivalently, each player must solve the optimisation problem:

(10) $$\max_{s_i \in S_i} u_i(s_i, s_{-i}^*)\,.$$

Applying this (generalised) optimisation to the Bertrand model implies that the price pair $(p_i^*,\ p_{-i}^*)$ is a Nash equilibrium if, for each firm $(i = 1{,}2)$, p_i^* solves:

(11) $$\max_{0 \le p_i < \infty} \Pi^i(p_i, p_{-i}^*) = \left(a - p_i + bp_{-i}^*\right)[p_i - c]\,.$$

The first-order condition for firm i is

(12a) $\quad \Pi_i^i = a - 2p_i + bp_{-i}^* + c = 0\,.$

Condition (*12a*) implies

(12b) $$p_i^* = \frac{1}{2}\left(a + c + bp_{-i}^*\right).$$

As a result, if $(p_1^*,\ p_2^*)$ is a Nash equilibrium, the firms' choices must satisfy:

(13a) $$p_1^* = \frac{1}{2}\left(a + c + bp_2^*\right)\text{, and}$$

(13b) $$p_2^* = \frac{1}{2}\left(a + c + bp_1^*\right).$$

Given (*13a*) and (*13b*), the equilibrium outcome of the Bertrand duopoly game is

(14) $$p_1^* = p_2^* = \frac{a + c}{2 - b}.$$

Note that the demand function is unrealistic in the sense that if firm 1 charges an arbitrarily high price, demand is still positive for firm 1's product, provided that firm 2 charges a sufficiently high price as well. The problem is sensible only if $b < 2$.

III.B.3: Applications to Electricity Markets

The classic Bertrand model demonstrates that with i) product homogeneity and identical unit costs, ii) no capacity constraints, and with iii) the simultaneous choice of price by firms, significant competition occurs through which competitors have the incentive to undercut each other's prices vigorously, resulting in a competitive outcome. An important rationale for the application of the Bertrand model to electricity markets is that, given the non-storability property of electricity, it may be subject to significant short-run price competition [1]. The motivation is that as long as price exceeds marginal cost and producers have sufficient capacity to meet demand, they will undercut each other's prices in an effort to gain market share.

In the classic Bertrand model, price competition under constant returns to scale yields an equilibrium price equal to marginal cost. Producers in electricity markets typically face capacity constraints. In the presence of capacity constraints, the Bertrand model does not necessarily predict that firms charge competitive prices. The basic intuition is that if a firm increases its price slightly above the competitive price, it loses some demand. This outcome is only a second-order effect, however, because the firm experiences a first-order increase in its profit due to the higher price on the infra-marginal units sold. Consequently, in applying the Bertrand model to analyse price competition in electricity markets, it is important to account for capacity constraints, as well as for the rationing rule for demand, since the nature of these assumptions may affect the equilibrium outcomes.

Further, electricity is a network industry, with a transmission network serving both producers and consumers that are spatially dispersed across it. As a result, transmission costs, specifically congestion costs and resistance losses, contribute to the divergence of (total) marginal cost, *i.e.* marginal production cost plus marginal transmission cost, between producers in two different network locations. Although price differences across locations in a network may certainly result from scarcity of supply or transmission services, these differences may enable producers to price discriminate on a spatial basis, *i.e.* they use geography to increase price at a location, and price differences between two locations may not be entirely cost-based. (This idea is distinct from the strategic manipulation of transmission constraints by producers [7]).

In an examination of U.S. deregulation proposals, Hobbs [14] computes price equilibria for a network model of spatial oligopoly in which both supply and demand are dispersed. Applying the Bertrand assumption to competitor behaviour in an oligopolistic electricity market setting, Hobbs finds that spatial heterogeneity in production costs and demand allows producers to practice 'shadow' marginal cost pricing under certain conditions. With this marginal cost pricing, the 'low cost' producer sells at a price slightly less than the marginal cost of its 'closest' competing neighbour, where 'closest' is used in the context of cost. Consequently, Hobbs demonstrates that in the presence of capacity constraints and transmission costs, spatial price discrimination in an electricity network leads to prices above the marginal cost of supply (including the marginal cost of transmission services).

The applicability of the Bertrand model to electricity markets depends on the specific modelling objectives and the market and institutional features that the model is intended to capture. Some of the economics/operations research supports the Bertrand model as a reasonable model of short-run price competition under certain conditions. In general, Bertrand competition may realistically represent firm behaviour in an electricity market over ranges of generator output/operation where competing firms' marginal costs are relatively 'flat' and excess capacity exists (these two factors may be correlated). The relevance of the Bertrand model may be more questionable when competing firms face 'high' demands and significant capacity constraints exist. These conclusions, however, are generalisations, and justifications for the use of the model must be analysed on a case-by-case basis, given the specific market context.

III.C: Cournot

III.C.1: Description

In the classic model of Cournot duopoly, firms compete against each other using quantities as strategy choices [8]. Three basic assumptions set the stage for traditional Cournot competition. First, competing firms produce homogeneous products and have identical unit production costs. Second, firms 'meet only once' in the market and choose quantities non-cooperatively and simultaneously. The non-cooperative nature of the game implies that firms act in their own self-interest, and the simultaneous aspect implies that each competing firm chooses its quantity without observing the choices of other firms. Third, given competing firms' choices of quantities, an hypothetical 'auctioneer' chooses the price that equates demand and supply. Variants of the classic model vary assumptions on product homogeneity, identical firm unit costs, *etc*. In the Cournot model of duopoly, each firm correctly anticipates its rival's quantity choice, such that the quantity it chooses in equilibrium maximises its profit (given the quantity choice by its rival).

Assume that two firms (i, j) are duopolists that produce an identical product. Firms i's profit function is

$$(15) \qquad \Pi^i(q_i, q_j) = P(q_i + q_j)q_i - C_i(q_i),$$

where P is the market demand function, and C_i is firm i's cost function. Assume that firm i's profit function is concave in q_i and twice differentiable. The first-order condition for profit maximisation is

$$(16) \qquad \Pi_i^i = \left[P(q_i + q_j) - C_i'(q_i)\right] + q_i P'(q_i + q_j) = 0.$$

The bracketed term [ ] in (16) is the price-cost margin, and it gives the profitability of an extra unit of output to firm i. The last term captures the effect of an extra unit of output on the profitability of inframarginal units of output, *i.e.* the production of an extra unit of output decreases the price by P () on the q_i units already produced by the firm. This term also illustrates the negative externality that exists between the two Cournot duopolists. Specifically, when selecting its output, firm i only accounts for the effect of the price change on its own output (q_i), not on the industry output ($q_i + q_j$). As a result, firms choose output levels in equilibrium that exceed (in aggregate) the optimal level from the perspective of the entire industry. The solution to the classic Cournot model, therefore, produces an equilibrium price that is less than the monopoly price but greater than the competitive price.

Further, a firm's output decreases with its own marginal cost but increases with its competitor's marginal cost. This feature of Cournot competition occurs because a relatively higher C_j leads firm j to reduce its output, which increases the residual demand to firm i. This conclusion is obtainable for general demand and cost functions provided that: i) firms' reaction functions are downward-sloping, *e.g.* quantities are strategic substitutes, and ii) firms' reaction functions intersect with each other only once, *i.e.* there exists a unique Cournot equilibrium (see Appendix A).

Kreps and Scheinkman [K-S] analyse the Cournot model in the context of *ex ante* investment and *ex post* price competition [16]. Specifically, [K-S] examine a two-period game with capacity constraints in which two firms simultaneously choose their capacities in the first period and then simultaneously choose prices - within their capacity limits - in the second period. They show that if the demand function is concave and if the rationing rule is efficient then the outcome of this game is equivalent to a one-period game in which firms simultaneously choose quantities and an auctioneer determines the market-clearing price, *i.e.* that this two-period outcome is precisely the Cournot outcome. This outcome occurs because the competition in the first period reduces capacities to such an extent that second period price (Bertrand) competition is effectively pre-empted. The result is important because it implies that 'quantity competition' is really a choice of scale that yields competing firms' cost functions, which determine the conditions of price competition. In addition, critics have often faulted the Cournot model because firms, not an auctioneer, ultimately choose prices. The [K-S] two-stage model to a large extent vindicates the Cournot model because it effectively subsumes price competition.

An important issue with the Cournot model is its handling of the price formation process. Since Cournot competitors choose quantities, there is no explicit representation of a bidding process. Prices 'fall out' of the demand function based on the aggregate Cournot quantities of the competitors in the market. Further, Cournot rivals do not respond to price changes; therefore, Cournot results are sensitive to both the form of the demand function and the demand elasticity. In general, a given firm will not have an unlimited scope to push prices increasingly higher without a response from its rivals.

In general, the Cournot model is a popular equilibrium model of imperfect competition in the economics literature for several reasons. First, the Cournot solution yields a negative correlation between the number of firms in the industry and profitability (see Appendix A). This correlation appears consistent with observations of market concentration in many industries. Second, the strategy by which firms withhold output in order to increase the market price above marginal cost appears to represent behaviour in some real world oligopoly markets.

III.C.2: Example

The classic Cournot game of duopoly postulates that two duopolists in a market interact with each other through quantity competition. One version of the game assumes that:

- two firms ($i = 1{,}2$) produce quantities (q_i) of a homogeneous product;
- total cost of production of q_i for firm i is $C_i(q_i) = cq_i$, where marginal cost is constant at c, with $c < a$;
- inverse demand for the product is $P(Q) = a - Q$ for $Q < a$ and P is the market price with $P(Q) = 0$ for $Q \quad a$, and $Q = q_i + q_{-i}$; and that
- firms simultaneously choose their respective output (q_i) from the feasible set $q_i = [0, \quad)$ and sell it at the market-clearing price.

These assumptions give all of the required components of the strategic form. First, the players are the two firms that comprise the duopoly. Second, the strategies available to each firm are the feasible quantities, and it is assumed that the outputs are infinitely divisible. Third, each firm's strategy space is denoted $S_i = [0, \quad)$, implying that a strategy satisfies $q_i \quad 0$. It is arguable that extremely large quantities, *e.g.* $(\infty - \varepsilon)$, should not be feasible; however, neither firm will produce $q_i > a$ since $P(Q) = 0$ for $Q \quad a$. Finally, it is necessary to specify the pay-off function of firm i as a function of its strategy and the strategy selected by the rival firm. In the Cournot duopoly, a firm's pay-off is described by its profit function. The generalised pay-off to firm i, $u_i(s_1, s_2)$, applied to this two-player strategic form game, therefore, is specified as:

$$(17) \qquad {}^{i}(q_i, q_{-i}) = q_i[P(q_i + q_{-i}) - c] = q_i[a - (q_i + q_{-i}) - c].$$

Recall that the strategy pair, (s_1^*, s_2^*), is a Nash equilibrium if, for each player i,

$$(18) \qquad u_i(s_i^*,\, s_{-i}^*) > u_i(s_i\,,\, s_{-i}^*)\ ,\ \forall s_i \in S_i.$$

Equivalently, each player must solve the optimisation problem:

$$(19) \qquad \max_{s_i \,\in\, S_i} u_i(s_i, s_{-i}^*)\,.$$

Applying this (generalised) optimisation to the Cournot model implies that the quantity pair (q_i^*, q_{-i}^*) is a Nash equilibrium, if for each firm ($i = 1,2$), q_i^* solves:

$$(20) \qquad \max_{0 \leq q_i < \infty} \Pi^i(q_i, q_{-i}^*) = q_i\left[a - (q_i + q_{-i}^*) - c\right]$$

If $q_{-i}^* < (a - c)$, the first-order condition for firm i's optimisation problem is both necessary and sufficient (shown later). The first-order condition for firm i is

$$(21a) \qquad \Pi_i^i = a - 2q_i - q_{-i}^* - c = 0\,.$$

Condition (*21a*) implies

$$(21b) \qquad q_i^* = \frac{1}{2}(a - c - q_{-i}^*)\,.$$

As a result, if (q_1^*, q_2^*) is a Nash equilibrium, the firms' choices must satisfy:

$$(22a) \qquad q_1^* = \frac{1}{2}\left(a - q_2^* - c\right),\ \text{and}$$

$$(22b) \qquad q_2^* = \frac{1}{2}\left(a - q_1^* - c\right).$$

Given (*22a*) and (*22b*), the symmetric equilibrium outcome of the Cournot duopoly game is

$$(23) \qquad q_1^* = q_2^* = \frac{a - c}{3}.$$

The solution value is less than ($a - c$), confirming that the first-order condition is both necessary and sufficient.

Each firm would like to be a monopolist in this market. In this case, a firm would choose q_i to maximise $\ ^i(q_i,0)$, producing the monopoly quantity, $q_m = (a - c)/2$ and earning the monopoly profit, $\ ^m(q_m,0) = (a - c)^2/4$. Since there are two firms in the market, however, duopoly profits are maximised by setting joint output ($q_1 + q_2$) equal to the monopoly output, q_m, which occurs if each firm produces the duopoly output, $q_i = q_m/2$. These output levels, however, are not an equilibrium because each firm has an incentive to increase output above $q_m/2$, since the monopoly price, $p_m(q_m/2 + q_m/2)$ is high. Of course, such behaviour drives the market price lower. This incentive to deviate can be verified by checking mathematically that $q_1 = q_m/2$ is not firm 1's best response to firm 2's output, $q_2 = q_m/2$.

III.C.3: Applications to Electricity Markets

Applications of the Cournot model to electricity markets assume several possible forms. The most common application is based on the assumption that individual firms, *i.e.* generation companies, do not alter their output levels given the output levels of their rivals. Other applications are that firms do not change their sales to a service region in response to the sales of other firms, and firms take the transmission flows on the network that other firms' generation creates, as given in their decision-making.

The [K-S] interpretation of the original Cournot framework is not uncommonly cited in the literature to motivate the use of the Cournot model in an electricity market context since producers in electricity markets often face capacity constraints. The [K-S] results, however, may not be fully applicable in this setting because the results are sensitive to the type of rationing that occurs during the second period [9]. Since electricity is non-storable and varies with time, *e.g.* off-peak versus peak demand, if generators withdraw capacity during peak times, it is unclear that such capacity limitation will affect non-peak periods. Consequently, citing the [K-S] results to justify the use of the Cournot model in an electricity market context - without some distinction between the periods of demand to which the model may be more realistically applicable, *e.g.* off-peak versus peak - is debatable.

Other motivations for use of the Cournot model include its conceptual simplicity and its computational flexibility. Specifically, given that POOLCO markets essentially use an ISO-based auction to match bids and offers to clear the market, the hypothetical auctioneer, which determines the market-clearing price in Cournot models, is conceptually appealing. From a computational perspective, the Cournot model is also appealing because equilibria can be obtained from both simple and complex models without overly restrictive assumptions on functional forms, *etc.*

The sensitivity of the Cournot results to the form of the demand function and to the demand elasticity has important implications for modelling electricity markets. Since low or near-zero price elasticities of demand are associated with the short-run demand for electricity and ancillary services, the Cournot model may predict unrealistically (and possibly irrelevant) equilibria under certain conditions. Given these issues, the 'appropriate' demand representation in a Cournot model of an electricity market is not necessarily clear.

The applicability of the Cournot model to electricity markets depends on the specific modelling objective(s) and the market and institutional features that the model is intended to capture. In general, Cournot competition may realistically represent firm behaviour in an electricity market over ranges of generator output/operation where competing firms' marginal costs are relatively 'steep' and capacity constraints exist (these two factors may be correlated). The presence of capacity constraints should not necessarily be the exclusive motivation for use of the Cournot model. These conclusions are generalisations, however, and justifications for the use of the Cournot model must be analysed on a case-by-case basis, given the specific market context.

III.D: Supply Function Equilibrium

III.D.1: Description

The third model for the analysis of imperfect competition is the supply function equilibrium model (SFE), in which firms compete with each other through the simultaneous choice of supply functions [15]. Klemperer and Meyer developed SFE in order to model competition in the presence of demand uncertainty. The idea behind their model is that even if an oligopolist knows its competitors' outputs, the presence of demand uncertainty implies that the oligopolist faces many possible demand profiles. Accordingly, management's decisions about the size, structure, corporate values, and decision rules of the firm implicitly determine a supply function that identifies the outputs that the firm will sell at prices that the market will accept. Such a supply function provides the firm with flexibility in adapting to changing business conditions, *e.g.* demand uncertainty, which simpler strategies that commit to either fixed prices or quantities preclude.

The SFE model is more intuitively appealing than the Bertrand and Cournot models because it allows for a strategy space in which competing firms choose entire supply functions. The strategies of the Bertrand and Cournot models are limited because firms choose either prices or quantities. Consistent with the Nash equilibrium solution concept that the three models share, each firm's choice of a supply function occurs simultaneously. In general, SFE price equilibria are generally between the Bertrand and Cournot extremes.

The 'intermediacy' of the SFE equilibrium results follows from the structure of the model relative to the Bertrand and Cournot models. Specifically, firm 2's choice of a supply function impacts firm 1's residual demand and *vice versa*. Given an increase in the price, two effects occur: i) the quantity demanded by the market decreases and the ii) quantity supplied by firm 2 increases. These effects make firm 1's residual demand function more price-sensitive than the market demand function. Consequently, supply function competition causes each firm to make its rival's residual demand function more sensitive to price relative to the market demand.

In contrast, the Cournot and Bertrand models are limiting cases. In the Cournot model, supply is completely unaffected by price, and the residual demand function is no less sensitive than the market demand function; the opposite is true for the Bertrand model. Equivalently, these models exogenously impose vertical supply curves on firms (Cournot) or horizontal supply curves on firms (Bertrand). By fixing either quantities or prices at given levels, these models force the other variable to accommodate all of the adjustments necessary to achieve a market-clearing outcome. In this sense, the Cournot and Bertrand models represent extremes, ranging from no response to a price increase by rivals (Cournot) to an infinitely large response by rivals (Bertrand).

This discussion naturally leads to the question of why firms would willingly choose supply functions, as opposed to quantities, as strategies if the supply function equilibrium results in a less profitable outcome for both firms. The answer is demand uncertainty. Suppose that two firms compete against each other in a market by choosing supply functions. The firms know the market demand curve is linear with a (fixed) slope, but they do not know its position, which is variable. Suppose that firm 1 is playing its optimal strategy and firm 2 correctly anticipates firm 1's strategy choice. Firm 2's optimal strategy then depends on the level of demand: if demand is 'high' then firm 2 should bid a high quantity to maximise its profit, and likewise, if demand is 'low' then firm 2 should bid a low quantity to maximise its profit. Firm 2 can implement this strategy by bidding a supply function that specifies a low quantity if the price is 'low' and a high quantity if the price is 'high'. The same exercise applies to firm 1.

An important interpretation of the SFE results is that they indicate the conditions under which the Cournot and Bertrand models approximate oligopolistic competition. Quantity-setting (Cournot) models may be more appropriate than price-setting (Bertrand) models if the number of firms is small, products are differentiated, demand certainty is additive, or marginal cost is 'steep' relative to demand. Alternatively, price-setting models may be more appropriate than quantity-setting models if the number of firm is large, products are more homogeneous, demand uncertainty is higher at lower prices, or marginal cost is 'flat' relative to demand.

A major weakness in the SFE model is that equilibria are difficult to calculate without restrictive assumptions on the number of firms and the form of firm costs, capacity constraints, and bid (supply) functions. Further, there may be no equilibrium, or multiple equilibria may exist. In the latter case, critics contend that the likelihood of multiple equilibria gives the model poor predictive value.

III.D.2: Applications to Electricity Markets

Green and Newbery [12] first modified the SFE model to represent supply function bidding in a POOLCO-based electricity market. In order to make the analysis tractable, they make several simplifying assumptions: i) each firm submits a 'smooth' (as opposed to step) supply function that relates quantity to price, and ii) firms do not receive a 'start price' each time they initiate generation (inclusion of a start price introduces a non-convexity into the problem). They first examine the case of a symmetric duopoly and find a range of possible equilibria and demonstrate that the inclusion of generator capacity constraints in the problem reduces the range of equilibria. Green and Newbery then examine the case of an asymmetric duopoly. In the asymmetric case, the larger firm tends to choose a more inelastic supply function (relative to the symmetric case) since it tends to gain more from a price increase. This action gives the smaller firm a relatively inelastic residual demand and an incentive to increase its price in turn. The net effect is to create a more inelastic industry supply curve. Green and Newbery then simulate the U.K. spot market under the industry assumption of a symmetric duopoly.

The SFE model may serve as a more realistic approach to modelling imperfect competition in certain markets. SFE strategies are equivalent to the submission of production schedules or offer curves that explicitly map a set of prices to a set of associated quantities. Consequently, the SFE model matches well with a centralised market-clearing mechanism, such as a POOLCO, that utilises an ISO-based auction process in which each generator offers a supply function that conveys an amount of capacity that it is willing to make available to the market at a specific price. Electricity markets may be the most realistic example of a market to which the SFE model is applicable [4,12].

A major obstacle to the realistic application of SFE to models of electricity networks is computational difficulty. Typically, SFE studies are designed for simple systems, *e.g.* three or four nodes. With the representation of large transmission networks and many generators subject to capacity constraints, a generator's optimisation problem is often non-convex and may yield multiple, local optima. In these cases, it is necessary to use strong limitations on the form of the bid functions, *e.g.* a linear bid function with only the slope variable; otherwise, it becomes necessary to impose unrealistic assumptions, *e.g.* firms possess identical marginal cost functions.

IV. Appendix A: Technical Notes on the Cournot Model

IV.A: Concavity of the Firm's Objective Function

Since firm i's objective function contains only a single choice variable (q_j is a parameter), concavity simply requires that $\Pi^i_{ii} < 0$, which implies:

$$(A1) \qquad \Pi^i_{ii} = 2P' + q_i P'' - C''_i < 0\,.$$

Condition ($A1$) is satisfied if the inverse demand function is concave ($P'' \leq 0$), and the cost function is convex ($C''_i > 0$). These assumptions, for example, are satisfied for linear demand functions ($P'' = 0$) and constant returns to scale technologies ($C''_i = 0$).

In addition, note that the concavity of the demand function is sufficient for quantities to be strategic substitutes ($\Pi^i_{ij} < 0$) since $\Pi^i_{ij} = P' + q_i P''$ and $P' < 0$.

IV.B: Uniqueness of Equilibrium

Given that a pure strategy Nash equilibrium exists for the Cournot model, such an equilibrium may not necessarily be unique. Multiple equilibria occur when firms' reaction functions intersect more than once. The attainment of an unique equilibrium requires that firms' reaction functions:

- intersect only once; and that they
- satisfy an asymptotic stability condition.

For purposes of examining the condition for asymptotic stability, consider the two-firm case. Assume that each firm's profit function is strictly concave in its own output. Recall that firm i's first-order condition for profit maximisation is:

$$(A2) \qquad \Pi^i_i(R_i(q_j), q_j) = 0\,.$$

The slope of the reaction function for firm i is:

$$(A3) \qquad \left|R'_i(q_j)\right| = \left|\frac{\Pi^i_{ij}(R_i(q_j), q_j)}{\Pi^i_{ii}(R_i(q_j), q_j)}\right|,$$

where $|\ |$ indicates the absolute value.

A sufficient condition for asymptotic stability of an equilibrium is that

$$(A4a) \quad \left|R'_i(q_j)\right|\left|R'_j(q_i)\right| < 1\,,$$

which implies,

$$(A4b) \quad \left(\Pi^i_{ij}\right)\left(\Pi^j_{ij}\right) < \left(\Pi^i_{ii}\right)\left(\Pi^j_{jj}\right).$$

IV.C: Industry Size and Profitability

In the Cournot model, industry size and profitability are inversely related. For purposes of exposition (and without loss of generality), assume:

- an industry comprises $i = 1,\ldots,n$ identical firms that behave as Cournot players in the market for an identical product;
- the linear market demand facing the industry is $P(Q) = 1 - Q$, where Q represents the aggregate output of the firms; and
- all firms have the identical cost structure $C_i(q_i) = cq_i$, with $c < 1$.

A representative firm (i) solves the following profit maximisation problem:

$$(A5a) \qquad \max_{0 \leq q_i < \infty} \Pi^i(q_i, \hat{q}_{-i}) = [\,P(Q) - c\,]\,q_i\ ,$$

where $\hat{q}_{-i} = (q_1,\ldots,q_{i-1},q_{i+1},\ldots,q_n)$. Since all rival firms are identical, their outputs will be the same; therefore, let $q = q_1 = \ldots = q_{i-1} = q_{i+1} = \ldots = q_n$. Rewriting condition $(A5a)$ gives:

$$(A5b) \qquad \max_{0 \leq q_i < \infty} \Pi^i(q_i, \hat{q}_{-i}) = [\,1 - (q_i + (n-1)q) - c\,]\,q_i\ .$$

The first-order condition for profit maximisation is

$$(A6) \qquad \Pi_i^i = 1 - 2q_i - (n-1)q - c = 0\,,$$

which is equivalent to

$$(A7) \qquad \Pi_i^i = 1 - q_i - Q - c = 0\,.$$

Using the fact that firm i is also identical to the other firms ($q_i = q$) yields the equilibrium output (q^*) for each firm:

$$(A8) \qquad q^* = \frac{1-c}{1+n}\,.$$

Given the demand function, the market price is

$$(A9) \qquad P = 1 - \left[\frac{n(1-c)}{1+n}\right],$$

which is more usefully rewritten as

$$(A10) \qquad P = c + \left(\frac{1-c}{1+n}\right).$$

Firm i's profit, therefore, is

$$(A11) \qquad \Pi^i = \left(\frac{1-c}{1+n}\right)^2.$$

From (*A11*), each firm's profit decreases with the number of firms in the industry. From (*A10*), the market price also decreases with the number of firms in the industry, and it follows that the aggregate (industry) profit, n , decreases as well. As a result, for a very large number of firms (n), the market price approaches the competitive price, *i.e.* $P = c$. Consequently, a Cournot equilibrium with a very large number of firms approximates a competitive equilibrium.

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